VOLUME XL NUMBER 1

THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

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JULY 1914

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

AGENTS:
THE CAMBRIDGE UNIVERSITY PRESS, LONDON AND EDINBURGH
KARL W. HIERSEMANN, LEIPZIG
THE MARUZEN-KABUSHIKI-KAISHA, TOKYO, OSAKA, KYOTO

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ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOL. XL. CONTENTS FOR JULY 1914 NO. 1

The Astrophysical Journal is published during each month except February and August. ¶ The subscription price is \$5.00 per year; the price of single copies is 65 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶ Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total \$5.30), on single copies, 3 cents (total 68 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total \$5.62), on single copies, 11 cents (total 76 cents). ¶ Remittances should be made payable to The University of Chicago Press, and should be in Chicago or New York exchange, postal or express money order. If local check is used, 10 cents must be added for collection.

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Business correspondence should be addressed to The University of Chicago Press, Chicago, Ill.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, The University of Chicago, Chicago, Ill.

Entered January 17, 1895, at the Post-office at Chicago, Ill., as second-class matter, under act of Congress, March 3, 1879

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RADIAL MOTION IN SUN-SPOTS?

By W. H. JULIUS

In two elaborate and interesting papers, bearing the above title without the interrogation point, St. John[1] has very skilfully discussed his observations on line displacements in sun-spot spectra from the point of view that the efficient cause of those displacements is motion in the line of sight. Although at the end of the second paper he also devotes a few pages to criticizing the theory that attempts to explain such phenomena on the basis of anomalous dispersion, he could not, of course, do full justice to a point of view differing so radically from his own. I therefore thought it my duty to defend the attacked position which is by no means so weak as he represents it.

THE INTERPRETATION BASED ON THE DOPPLER EFFECT

St. John's investigation relates to eleven spots in positions between 25° and 60° from the center of the disk. The slit of the spectrograph was parallel to the radius of the solar image, passing through the center of the spot umbra. Nearly all of the 506 lines included in the measurements are displaced to the red on the edge of the penumbra nearest the limb, and to the violet on the opposite edge; only 13 lines show the reverse effect. In agreement with

[1] *Mt. Wilson Contr.*, Nos. 69 and 74; *Astrophysical Journal*, **37**, 322, and **38**, 341, 1913.

Evershed, who discovered the phenomenon in 1909, St. John concludes that the displacements are due to radial flow of matter tangential to the solar surface, generally directed from the center of the spot outward, but, on the contrary, inward in the case of the 13 exceptional lines.

If this interpretation of the displacements were free from serious difficulties it would be unnecessary to propose other views of the subject. It is a fact, however, that there are great difficulties; hence the pro and con of rival theories may well be submitted to the consideration of astrophysicists.

The displacements on which St. John's conclusions depend are of the order of magnitude 0.020 A, only very few exceeding 0.040 A. It certainly testifies to that author's experimental proficiency as well as to the power of the Mount Wilson apparatus that on such minute quantities a regular investigation can be based; but at the same time the conditions of the research warn us against putting too much confidence in the individual observations. On p. 4 of *Contribution No. 69*, St. John explains why the mean deviations (given in the sixth column of his Table I) from the mean displacements calculated for every line are so considerable. Trustworthy results can therefore be expected only from those considerations in which the statistical treatment includes a sufficiently great number of observed displacements, whereas the reliability of apparent relations rapidly decreases with the number of measurements from which they are deduced.

A survey of the whole series of measurements led St. John to the discovery of two very important and striking laws: (1) the displacements increase with increasing wave-length; (2) the displacements progressively decrease with the increase of line intensity.

Tables II and III,[1] illustrating these laws as they appear in the measurements on 193 iron lines of intensity 1 to 8, are reproduced here for convenience of reference. In the upper section of Table II the displacements are as measured; in the lower section they are reduced to a common wave-length, λ 5000. This reduction has been applied by St. John on the ground that, if the displacements

[1] *Mt. Wilson Contr.*, No. 69, pp. 16–17; *Astrophysical Journal*, **37**, 337–338, 1913. I keep the numbers II and III for these tables, so that in the present paper there is no Table I.

are due to the Doppler effect, they should be proportional to the wave-length. He therefore divides every displacement by the fraction $\lambda/5000$; the values thus obtained form the lower section of the table; they should be equal for lines of equal intensity.

TABLE II

DISPLACEMENTS AND WAVE-LENGTH

Region	Mean λ	Intensity								Mean
		1	2	3	4	5	6	7	8	
Violet.	4017	0.022	0.020	0.014	0.014	0.013	0.011	0.008	0.006	0.014 A
Yellow-red.	6121	.032	.030	.032	.024	.028	.024	.019	.016	.026
Violet.	4017	.026	.024	.019	.018	.017	.013	.011	.008	.017
Yellow-red.	6121	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	0.021

TABLE III

DISPLACEMENTS AND LINE INTENSITIES REDUCED TO λ 5000

Mean λ	Intensity								No Lines	Mean Interval
	1	2	3	4	5	6	7	8		
4017.	0.026	0.024	0.019	0.018	0.017	0.013	0.011	0.008	81	0.003 A
4992.	0.030	0.026	0.026	0.025	0.018	0.016	0.006	0.007	69	0.003
6121.	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	43	0.002
Wtd. mean.	0.028	0.025	0.023	0.021	0.019	0.016	0.012	0.009	. . . . . .	0.003
Vel. km/sec.	1.68	1.50	1.38	1.26	1.14	0.96	0.72	0.54	. . . . . .	0.18

Now, I think it impossible to agree with St. John when he concludes that the results collected in Table II "seem decisively in favor of an effect varying as the wave-length" and that "the decrease from 0.012 A to 0.004 A in the mean difference [between the average displacements in the violet and in the yellow-red regions] when the displacements are reduced to a common wave-length indicates that the observed differences are due to the Doppler effect, and that we are dealing with real movements of gases in the reversing layer." The table gives plenty of evidence, on the contrary, that the displacements are by no means proportional to the wave-length. Indeed, in every intensity class they appear to increase more rapidly than the wave-length. Moreover, the rate

of increase with wave-length is far from regular, for if we transfer the series of displacements relating to mean λ 4992 from Table III to the upper section of Table II and put it between violet and yellow-red, we see that for lines of low intensity the rate of increase of the displacements is in general more rapid between mean λ 4017 and λ 5000 than between λ 5000 and mean λ 6121, whereas for lines of high intensity the reverse seems to obtain.

There would not be any objection, of course, to advancing the *hypothesis* that the Doppler effect is involved in the phenomenon, and to try additional hypotheses in order to account for the systematic and the irregular deviations[1] from proportionality between displacement and wave-length. I deny only that the data included in the above tables give any decisive indication in favor of the view which considers the Doppler effect as the principal cause of the diplacements. Neither do the same data contain a positive proof that the rival explanation which is based on anomalous dispersion is the right one; but, as will be shown in the second section of this paper, the latter interpretation deserves notice and has more features that recommend it than the former if the value of both be judged by the criteria so well formulated by St. John at the beginning of the section on p. 14 of *Contribution No. 74*.

Reverting to St. John's discussion of the above tables, we remark that Doppler's principle alone has nothing in it to explain the remarkable and well-established progressive decrease of the displacements with the increase of line intensity. A new hypothesis, therefore, had to be introduced. St. John assumes that the weaker iron lines originate at the lower level, so that the displacements would increase with depth. By this assumption he intends to insure a *progressive* change in the velocity of outflow of matter with change of level, and at the same time opens a way to account for the fact that the displacements do not exhibit the otherwise expected proportionality with the wave-length, but increase more rapidly. For it might be that among the lines of the same intensity those belonging to the yellow-red region of the spectrum originate at a lower level than those belonging to the violet region, a difference which the author calls a natural consequence of the scattering of light by small particles.

[1] Cf. Fig. 6, p. 23.

Many spectroscopists, however, will shrink from accepting this bold hypothesis of the "Iron Scale" which supposes the various intensities of the solar lines of an element to indicate the levels where they originate. In the laboratory, the absorption spectrum of a gas (for instance, of NO_2) shows strong and weak lines at the same time; it is not easy to admit with St. John[1] that in the sun "the absorbing centers effective for lines of different intensities of a given element are apparently not identical and appear to be allocated in successive spherical shells." What are we to think, e.g., about the theories of series lines from this point of view?

But let us pass this difficulty, and see what the Doppler interpretation of these line-shifts results in.

All elements included in the investigation, except hydrogen, are moving from the axis of the spot vortex outward with velocities varying between 1.1 and 0 km per second. Hydrogen flows inward at a higher level, and so do special forms of calcium, magnesium, sodium, iron, and strontium, respectively represented by 3, 2, 2, 2, and 1 lines.

In general the horizontal velocities seem to be greatest near the outer edge of the penumbra. Multiplying the circumference of the spot by the distance between the lowest and the highest level of outflowing matter, we get the transverse section of the stream, in which the average velocity of the gases may be estimated at 0.7 km per second. The stream must be fed by a rising current in the center of the spot, the stream lines of which will progressively pass from a vertical to a horizontal direction. The vertical component of the motion would become the more conspicuous, the more a spot approaches the center of the solar disk; and as very probably the transverse section of the vertical current in the umbra is not much greater than that of the horizontal current near the outer edge of the penumbra, we should expect to find considerable displacements of most umbral lines toward the violet, especially in the spectra of spots located in the central parts of the disk. And in the penumbral spectra of eccentrically located spots the vertical component of the motion would have the effect of increasing the displacements to the violet and diminishing those toward the red.[2]

[1] *Mt. Wilson Contr.*, No. 74, p. 13; *Astrophysical Journal*, **38**, 353, 1913.

[2] The 13 exceptional lines are not considered here.

Definite indications of such vertical velocities are not mentioned, however. This, I think, is one of the most serious objections against the assumption that radial motion in sun-spots is the effective cause of the displacements in question. St. John suggests[1] that the levels where the vertical components of the velocities prevail are too low to be accessible to spectroscopic investigation. If this were true for the lines of intensity oo, it would not hold good for the lines belonging to higher levels, because their absorbing centers rise a long way through optically accessible layers. So the difficulty is not removed.

The opposite displacements of the 13 above-mentioned chromospheric lines are explained as indicating a radial inflow of chromospheric material into the spot. From the displacements which the lines H and K of calcium show in the spectrum of the umbra[2] it would follow that this gas is moving downward in the center of the spot with a velocity of 1.3 km per second. The other chromospheric gases are supposed to share the downward motion, although their lines appear not sufficiently displaced in the spectrum of the umbra to make the assumption plausible. The question where this chromospheric material goes after having reached the reversing layer finds no satisfactory solution in the results of the measurements; and it seems very difficult to reconcile the necessary consequences of an impact between the downward-rushing chromospheric matter and the upward-rushing matter of the reversing layer with the assumed relatively low temperature of the umbral region.

St. John attempts to corroborate his intensity-and-level hypothesis by considering the atomic weight of the elements in connection with the displacements of their lines in the spot spectrum. We may doubt whether the data suffice for the purpose. It is stated, e.g., that the lines of the heavy elements, such as barium, lanthanum, neodymium, cadmium, cerium, lead, and ytterbium, originate at lower levels than the lines of like intensity of iron;[3] but if we refer to Table I of the first paper we find that

[1] *Mt. Wilson Contr.*, No. 69, p. 24; *Astrophysical Journal*, **37**, 345, 1913.

[2] Cf. *Mt. Wilson Contr.*, No. 54, pp. 28–29; No. 69, pp. 24–25 and 27–28; *Astrophysical Journal*, **34**, 136–137, 1911; **37**, 345–346, 348–349, 1913.

[3] *Mt. Wilson Contr.*, No. 74, p. 6; *Astrophysical Journal*, **38**, 346, 1913.

the evidence is not so very strong. Lead, ytterbium, and cadmium are represented by only one line each, barium by two lines; these five lines really show larger displacements than the mean iron lines of like intensities. With lanthanum the conclusion has somewhat greater weight, because nine lines have been measured. Four of them (intensity 1) give the mean displacement 0.029 instead of 0.028 which would correspond to the iron scale; four lines of intensity 2 give 0.026 instead of 0.025, and one of intensity 4 gives 0.025 instead of 0.021. The differences are small (considering the great atomic weight of lanthanum), but in the required direction. With cerium, on the other hand, one of the two lines by which it is represented in the table shows a displacement 0.026 instead of 0.023 but the other one (intensity 1) gives 0.017 instead of 0.028 and thus points to a *higher* level. Of neodymium, finally, three lines have been measured; two of them (intensity 1) give the mean value 0.025 instead of 0.028 and one (intensity 2) gives 0.022 instead of 0.025, so that the lines of this element (atomic weight 144) would seem to originate on the average at a higher level than the iron lines of like intensity. An equally unfavorable account must be given of zirconium (atomic weight 91), for the displacements of six of its seven lines in the list would also decidedly, from the point of view of St. John's hypothesis, indicate higher levels than those corresponding to the iron scale.

It is possible, though not probable, that the large deviations which the mean displacements of individual lines often show with respect to the mean value corresponding to their intensity class and spectral region find a sufficient explanation in the extreme difficulty of the measurements. If, however, the accuracy attained would permit of regarding such deviations as genuine, the interpretation of the displacements on the basis of the Doppler effect would be condemned. One could not reasonably admit the various absorption centers present in a gas-current at a certain level to have different proper velocities of outflow from the spot vortex.

THE INTERPRETATION BASED ON ANOMALOUS DISPERSION

As would appear from the preceding discussion of the radial-motion hypothesis, it is not superfluous to look for other ways of

explaining the relative displacements of the Fraunhofer lines at the limb- and center-edges of the penumbrae of eccentrically located spots.

In a previous paper[1] I suggested an explanation of the sun's edge and of the general distribution of the brightness on the solar disk, assuming the non-existence of a photosphere in the sense of the surface of a body or of a layer of clouds. The term "photosphere" is preserved; but in the new interpretation it only means a mathematical sphere constructed round the sun's center and having for its radius the distance between the center and the apparent edge of the disk. The gaseous condition of the solar atmosphere continues below the photosphere without any abrupt change in the physical or chemical properties of the mixture. The increase of the mean density and the variation of the mean composition are progressive.

Our sun-spot hypothesis advanced in 1909[2] considered the distribution of the light in a spot as chiefly produced by the refraction which photospheric light suffers when traversing a region where the optical density passes through a minimum (e.g., a solar vortex).

It has been doubted whether in the solar *atmosphere* differences of density, sufficient for imparting to the rays the important deviations required by the theory, really could exist. This objection will disappear if the new interpretation of the photosphere be accepted, because the present point of view permits of locating that region of minimum density somewhere below the photospheric level, in layers where sufficient gradients of optical density are sure to be found.

A rough estimate of the values which optical density gradients must have in order to produce an observable incurvation of average rays (for which the medium possesses small refracting power) has been given in a previous paper.[3] It should be remembered that in the present paper we are dealing with R-light and V-light,[4]

[1] *Astrophysical Journal*, **38**, 129, 1913.

[2] *Proc. Roy. Acad. Amst.*, **12**, 266, 1909; *Physikalische Zeitschrift*, **11**, 56, 1910.

[3] *Astrophysical Journal*, **38**, 135, 1913.

[4] Defined as waves lying respectively on the red-facing and the violet-facing sides of absorption lines and very near to them; cf. *Proc. Roy. Acad. Amst.*, **12**, 275, 1909; or *Physikalische Zeitschrift*, **11**, 63, 1910.

that is, with waves generally more strongly refracted than the average light of the spectrum.

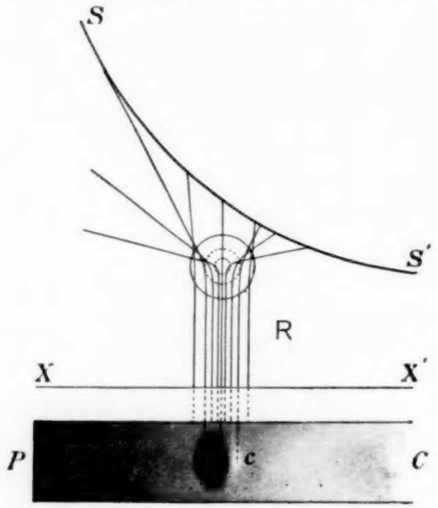


FIG. 1

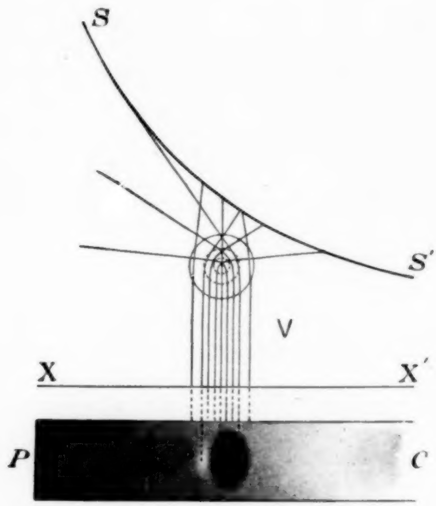


FIG. 2

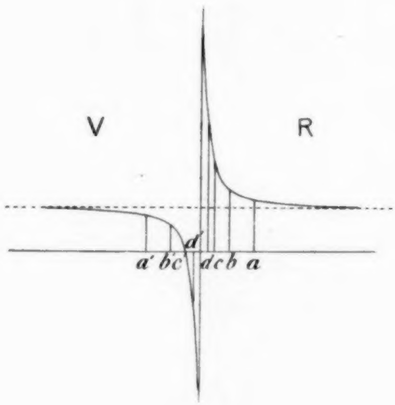


FIG. 3

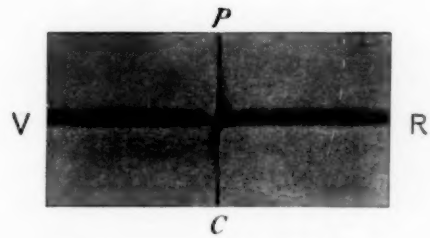


FIG. 4

The influence of an eccentrically located density minimum on the course of R-light and V-light, and the resulting effect on the spectrum, may be illustrated by means of Figs. 1, 2, 3, and 4. These are taken from the above-mentioned publication of 1909,

to which I refer for a more complete explanation of the purely *schematic* case represented by them.[1] It should be noted that the drawings were only intended to bring out the essential feature of the effect of anomalous refraction in an ideal depression. In a real spot the conditions are of course far more complicated; the shape of the depression will not be spherical as assumed, and irregular gradients superposed upon the systematic gradients of the vortex region will especially influence the course of the most refrangible rays, thus confusing the schematic results for the central parts of strong lines. But provisionally abstracting from details—to which we shall revert farther on—we see that anomalous refraction causes just the kind of wryness observed with all the lines of the spot spectrum, except with a few very strong lines that correspond to prominent chromospheric lines. A discussion of the behavior of those 13 exceptional lines of the table must be postponed; in this section we confine our attention to the normal case of the 493 lines.

Besides explaining the general character of the displacements in question, the anomalous-dispersion theory should account for the following quantitative results of the observations:

1. The *mean* displacements calculated for each intensity class decrease with increasing line intensity. (For explaining this rule St. John had to introduce his new hypothesis connecting line intensity with level.)

2. Large deviations from the mean values occur in every intensity class. (St. John does not consider such deviations as purely accidental, for he bases on them various conclusions on relative levels of different elements. But if the large residuals often found when deducting the mean from the individual displacements, for lines of one element and one intensity, are real, this

[1] Some experiments on the refraction of light in whirling gases were shown on the occasion of the fifth meeting of the Solar Union in August, 1913, at Bonn, and are described in *Physikalische Zeitschrift*, **15**, 48, 1913. As remarked in a footnote of that paper, our recent interpretation of the photosphere made it possible to improve the sun-spot hypothesis of 1909 by admitting that the spot vortex or region of minimum density might be located *below* the photospheric level. The arc *SS′* of our figures, therefore, does not now represent a part of the photosphere, but a part of a lower level.

would seem to present, as already remarked, an insuperable difficulty for the interpretation of the phenomenon as a radial-motion effect.)

3. The displacements increase with the wave-length, but not proportionally. The observations do not indicate a simple law, and the rate of increase seems to vary differently for different intensity classes. (It is impossible to explain these facts on the basis of the Doppler effect without calling in additional hypotheses in order to account for the large deviations from proportionality with wave-length.)

From the point of view of the anomalous-dispersion theory it is at once clear that there must be a direct connection between the intensity of the lines and the magnitude of their displacements in spots, as both phenomena depend on the "dispersion bands" enveloping the absorption lines. It would be rash and erroneous, however, to conclude that strong anomalous dispersion, because it produces Fraunhofer lines of great intensity, must also give rise to large relative displacements in the spot spectrum. Indeed, the reverse is true. This will come out clearly in the course of the discussion; but before settling this point we had better first consider the possible cause of the second rule: the great disparity of the displacements.

Disparity of the displacements. Mutual influence of Fraunhofer lines.—If the displacement of a certain line A depends on the refracting power of the medium for the adjacent waves, it must be influenced by the presence of a strong neighboring line B. Let us discuss the nature of that influence.

Let n_0 be the value which the refractive index would have in the part of the spectrum under observation if this were free from absorption lines, and suppose n_0 to be >1. The effect of a line B (Fig. 5) is to reduce the indices on its violet side and to raise them on its red side, as indicated by the partly broken curves. The line A, if isolated, would produce its own anomaly in the dispersion-curve as shown in A_1. If A were situated near B, in one of the positions A_2 or A_3, that anomaly would have a somewhat different shape in consequence of its being superposed upon one of the branches of the dispersion-curve due to B. The refracting

properties of the medium, being determined by the values of $n-1$, will be different in the three cases represented by A_1, A_2, and A_3.

Only those waves for which the absolute values of $\pm(n-1)$ exceed a certain minimum value will become sufficiently curved in the outer parts of the vortex region to give rise to sensible refraction effects in the spectrum of the penumbra. This is indicated in the figure by means of the two broken lines drawn at equal distances above and below the line $n=1$. We may assume that only the

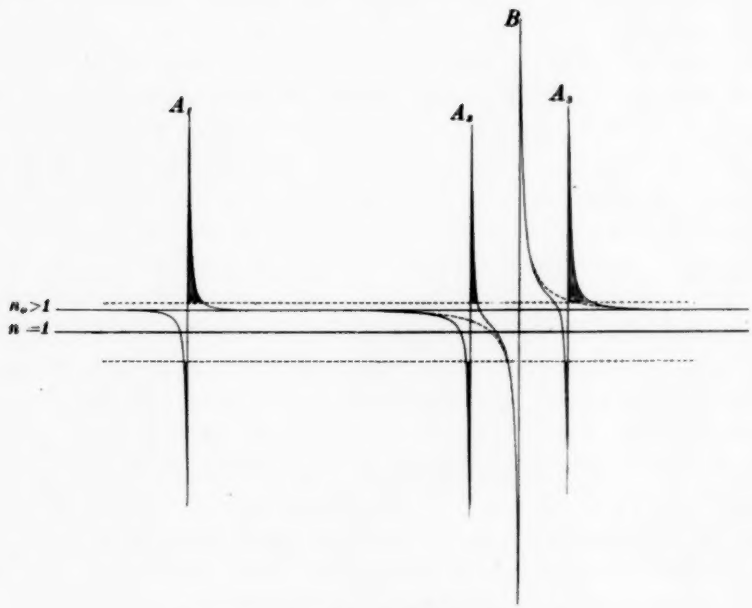


FIG. 5.—Mutual influence of Fraunhofer lines

parts of the dispersion-curve lying outside the zone between these broken lines are material to the formation of the dispersion bands enveloping the absorption lines of the penumbral spectrum. The R-light corresponding to the shaded area above the zone is responsible for the displacements toward the red observed at the peripheral edge of the penumbra; the V-light corresponding to the shaded area below the zone causes the displacements toward the violet at the central edge.

Now comparing for the lines A_1, A_2, and A_3 the horizontal distances between the "centers of gravity"[1] of their R-area and V-area, we at once realize that A_2 will show a smaller displacement than A_1, and A_3 a greater displacement than A_1; but it is also evident from the figure that the difference between the cases A_3 and A_1 is not so marked as that between A_2 and A_1.

This deduction from the theory can be put to the test by means of St. John's observations.

After having marked out on Higgs's atlas of the normal solar spectrum the 506 lines of Table I, I selected *all* cases in which a measured line A of intensity 3 or lower was on the *violet* side of a stronger line B (generally of intensity 4 or higher) at a distance of about 0.5 A or less. A few cases in which the line A had another strong companion B' equally near but on the other side were of course discarded. Forty-three pairs answering the conditions were found; they are united in the first column of Table IV. The wave-lengths of lines B not appearing in St. John's Table I were read on Higgs's atlas. Most of these lines could be identified with lines occurring in Mitchell's table of wave-lengths of the chromosphere.[2] In the second and third columns of our Table IV are the elements and intensities; the elements in brackets are taken from Mitchell's table. The fourth column contains the corrected values Δ' of the observed relative displacements as given by St. John.[3] In the fifth column are indicated "normal" values of the displacements. These were obtained from the data appearing in the upper section of Table II after having supplemented them by inserting between the numbers for violet and yellow-red those given for mean λ 4992 in Table III. Thus, on the basis of the two rules found by St. John that connect the displacements with line intensity and with wave-length, it was possible to indicate a "normal" displacement peculiar

[1] The term "center of gravity" is used for convenience' sake. Properly speaking, the estimated location of the lines will of course depend on considerations somewhat different from those involved in the determination of the center of gravity of the said area.

[2] *Astrophysical Journal*, **38**, 407, 1913.

[3] I have not used the values Δ' reduced to λ 5000, because that reduction derived its direct meaning from the assumption that the relative displacements are due to the Doppler effect.

TABLE IV

THE *Decrease* OF THE AMOUNT OF THE EVERSHED EFFECT IN THE SPECTRUM OF ECCENTRICALLY LOCATED SUN-SPOTS, OBSERVED WITH THE *Violet*-AND-WEAKER MEMBERS OF PAIRS OF LINES.

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
3649.137	Cr	1	0.014	0.022	−0.008	
3649.4		5				
3662.096	Ni	3	.015	.015	.000	
3662.38	(Ti)	5				
3686.926	Cr	1	.016	.022	− .006	
3687.2		3				
3687.234	Fe	3	.010	.015	− .005	
3687.610	Fe	6				
3688.210	V	1	.018	.022	− .004	
3688.5		4				
3690.599	Fe	2	.017	.020	− .003	
3690.8		3				
3707.702	Ti	2	.013	.020	− .007	
3708.07	(Fe)	6				
3708.964	Co	1	.015	.022	− .007	
3709.389	Fe	6				
3895.119	Co	3	.012	.015	− .003	
blend		4?				
3895.583	Mn	3	.008	.015	− .007	
3895.803	Fe	7				
3898.032	Fe	3	.007	.015	− .008	
3898.2		4				
3899.171	Fe	3	.013	.015	− .002	
3899.21	(V–Fe)	3				
3906.438	Co	2	.010	.020	− .010	
3906.628	Fe	10				
3913.123	Ni	2	.016	.020	− .004	
3913.609	Ti	5				
3916.879	Fe	5	.009	.014	− .005	
3917.32	(Fe)	6				
3947.522	?	2	.014	.021	− .007	
3947.675	Fe	4				
3956.603	Fe	4	.010	.014	− .004	
3956.879	Fe	6				
3958.073	Co	2	.018	.022	− .004	
3958.36	(Zr–Ti)	4				
3962.995	Ti	3	.012	.016	− .004	
3963.2		4				
3995.899	La	1	.014	.023	− .009	
3996.14		3				
3997.115	Fe	2	.019	.022	− .003	
3997.547	Fe	4				
4035.752	Co	2	.016	.022	− .006	
4035.883	Mn	4				
4109.609	Nd	1	.016	.024	− .008	
4109.95		4				
4132.100	V	2	0.011	0.022	−0.011	
4132.235	Fe	10				

TABLE IV—*Continued*

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
4133.755	Fe	2	0.022	0.022	0.000	
4133.95	(Fe–Ce)	4	....	....	....	
4149.360	Zr	2	.016	.022	− .006	
4149.5	....	4	....	....	....	
4216.136	CN	1	.022	.024	− .002	
4216.35	(Fe)	4	....	....	....	
4233.328	Mn–Fe	4	.017	.017	.000	
4233.772	Fe	6	....	....	....	
4271.325	Fe	6	.009	.014	− .005	Distance >0.6 A
4271.934	Fe	15	....	....	....	
4274.746	Ti	2	.016	.023	− .007	
4274.958	Cr	7	....	....	....	
4289.525	Ca	4	.015	.019	− .004	
4289.885	Cr	5	....	....	....	
4294.936	Zr	2	.018	.023	− .005	
4295.29	Dy	6	....	....	....	
4302.353	Fe	2	.019	.023	− .004	
4302.692	Ca	4	....	....	....	
4315.138	Ti	3	.010	.021	− .011	
4315.262	Fe	4	....	....	....	
4408.364	V	2	.027	.024	+ .003	Influenced by
4408.54	(V)	4	....	....	....	λ 4407.8, int. 6
5168.832	Ni	1	.026	.028	− .002	
5169.16	(Fe)	7	....	....	....	
5188.863	Ti	2	.015	.026	− .011	
5189.0	....	3	....	....	....	
5226.707	Ti	2	.020	.026	− .006	
5227.0	....	4	....	....	....	
5250.385	Fe	2	.028	.027	+ .001	
5250.82	(Fe)	3	....	....	....	
5298.194	Cr	1	.021	.029	− .008	
5298.455	Cr	4	....	....	....	
5598.524	Fe	1	.019	.030	− .011	
5598.711	Ca	4	....	....	....	
5615.520	Fe	2	.025	.027	− .002	
5615.877	Fe	6	....	....	....	
5624.245	Fe	1	0.027	0.031	−0.004	
5624.77	(Fe)	4	....	....	....	
					Mean diff. −0.0051	

to the spectral region and the intensity of each measured line. With these normal values the observed values had to be compared.

As expected, the differences shown in the sixth column are negative. There is one distinct exception: λ 4408.364, for which the difference is +0.003. This line, however, has also a companion *B′* on the violet side (λ 4407.8 of intensity 6) that would work the

opposite way, so that perhaps the case ought to have been discarded although the distance between A and B' is a little greater than 0.5 A. On the average, the measured displacements of these forty-three violet members of pairs are as much as 0.0051 A smaller than the normal values. This result is in perfect harmony with our assumption that the displacements depend on the refracting power of the medium.

Additional evidence is obtained from Table V, which contains data similar to those of Table IV, but now relative to thirty-nine pairs, the weaker line A of which is on the *red* side of the stronger line B. In these cases the observed displacements of A should generally exceed the normal values, but the effect is expected to be less conspicuous than the reduction of the displacements on the violet side of lines B.

As a matter of fact, the differences in the sixth column are for the greater part positive. And examining on Higgs's atlas the environment of the 12 lines that gave negative deviations from the normal displacements, we find some cases where an additional strong neighboring line B' on the wrong side may be responsible for the discrepancy (e.g., λ 3947.918 might be influenced by λ 3948.25 of intensity 5, and λ 3949.039 by λ 3949.25 of intensity 3). The average increase of the relative displacements above their normal values, attributed to the violet companions of our thirty-nine lines, amounts to nearly +0.0015 A. Omitting the dubious cases we should have found the mean residual +0.0019 A.

It must be granted that the difference between the absolute values of the negative mean residual 0.0051 and the positive mean residual 0.0015 appears too great to be entirely accounted for by the inequality of the configuration of the shaded areas on the two sides of B, as represented in Fig. 5. The difference, however, may be partly due to a systematic observational error; for it is not improbable that the proximity of a strong line B causes the displacements of lines A to be underestimated. Allowing for this error—which of course would have the same sign on either side of B—we must reduce the observed negative and enlarge the positive mean residual by the same amount. Their absolute values thus

TABLE V

THE *Increase* OF THE AMOUNT OF THE EVERSHED EFFECT IN THE SPECTRUM OF ECCENTRICALLY LOCATED SUN-SPOTS, OBSERVED WITH THE *Red*-AND-WEAKER MEMBERS OF PAIRS OF LINES

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
3694.24	(Fe–Ni)	8				
3694.344	Yt	3	0.020	0.015	+0.005	
3694.344	Yt	3				
3694.576	La	1	.027	.022	+ .005	
3704.603	Fe	4				
3704.840	V	1	.016	.022	− .006	
3706.24	(Mn–Ti–Ca)	7				
3706.363	Fe	3	.017	.015	+ .002	
3711.364	Fe	4				
3711.552	Fe	3	.015	.015	.000	
3898.2	....	4				
3898.531	Mn	2	.014	.020	− .006	
3947.675	Fe	4				
3947.918	Ti	2	.013	.021	− .008	Influenced by
3948.925	Fe	4				λ 3948.25, int. 5
3949.039	Ca	1	.018	.022	− .004	Influenced by
3950.102	Fe	15(?)				λ 3949.25, int. 3
3950.497	Y	5	.013	.014	− .001	
3984.17	(Fe–Mn)	6				
3984.294	Mn	2	.021	.020	+ .001	
3989.912	Ti	4				
3990.011	Fe	3	.012	.015	− .003	
4018.25	(Mn)	7				
4018.420	Fe	3	.022	.015	+ .007	
4078.49	?	4				
4078.631	Ti	3	.017	.016	+ .001	
4079.4	....	5				
4079.570	Mn	3	.018	.016	+ .002	
4134.54	(V–Fe)	6				
4134.840	Fe	5	.014	.014	.000	
4161.68	(Ti)	5				
4161.961	Sr	1	.025	.024	+ .001	
4184.32	(Ti–Gd)	5				
4184.472	Ti	2	.022	.022	.000	
4196.35	?	4				
4196.699	La	2	.024	.022	+ .002	
4236.112	Fe	8				
4236.429	Ni	1	.024	.024	.000	
4240.64	(Zr–Ce–Fe)	4				
4240.872	Cr	1	.022	.024	− .002	
4338.084	Ti	4				
4338.430	Fe	1	.025	.024	+ .001	
4637.685	Fe	5				
4638.193	Fe	4	.027	.021	+ .006	
4667.626	Fe	4				
4667.768	Ti	3	.027	.023	+ .004	
4679.027	Fe	6				
4679.409	Ni	2	0.037	0.024	+0.013	

TABLE V—*Continued*

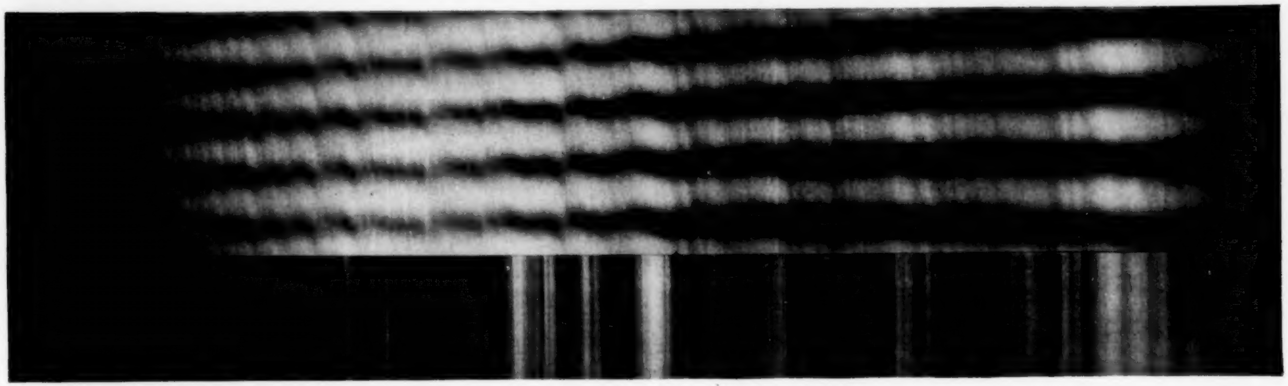
λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
4703.177	Mg	10				
4703.994	Ni	3	0.035	0.023	+0.012	Distance >0.6 A
4731.65	(Fe)	4				
4731.984	Ni	1	.030	.026	+ .004	
4736.96	(Fe)	6				
4737.540	Cr	2	.034	.024	+ .010	
4762.567	Mn	5				
4762.820	Ni	1	.039	.026	+ .013	
5129.42	(Ti–Ni)	5				
5129.546	Ni	2	.026	.024	+ .002	
5129.546	Ni	2				
5129.805	Fe	1	.033	.028	+ .005	
5131.642	(Fe–C)	3				
5131.942	Ni	1	.029	.028	+ .001	
5152.087	(Fe–C)	3				
5152.361	Ti	0	.031	.031	.000	
5192.523	(Fe–Nd)	5				
5193.139	Ti	2	.021	.026	− .005	Distance >0.6 A
5283.802	(Fe)	6				
5284.281	Ti	1	.026	.028	− .002	
5298.455	Cr	4				
5298.672	Ti	1	.022	.028	− .006	
5349.652	Ca	4				
5349.928	Fe	1	.027	.028	− .001	
5857.674	Ca	8				
5857.976	Ni	3	.030	.027	+ .003	
5953.0	(Ti–Fe)	5				
5953.386	Ti	1	.038	.032	+ .006	
6400.217	Fe	8				
6400.528	Fe	2	0.026	0.031	−0.005	
					Mean diff. +0.00146	

approach each other, whereas the characteristic difference between the cases A_2 and A_3 remains unaffected.

Taken all in all, the evidence is very strong in favor of the view that the displacements here considered are entirely due to anomalous refraction.

The interpretation of the phenomena on this basis easily accounts for the great and frequent deviations of individual displacements from the normal values, and thus increases our confidence in the accuracy of St. John's measurements. Indeed, if we take it for granted that every Fraunhofer line influences the refractive index of the gaseous mixture in a way similar to that in

PLATE I



Record obtained with a Jamin interferential refractometer and a Hilger spectrograph, showing how the refractive index of a selectively absorbing medium (NO_2) fluctuates along the spectrum under the influence of the absorption lines.

which our lines B have been proved to do, the value of n_0 must oscillate sensibly along the whole spectrum, especially in regions where the lines are crowded. The refracting power of the solar atmosphere would then be analogous to that of a terrestrial gas giving an absorption spectrum with a great number of lines. Plate I is intended to illustrate such a case. It shows the refracting power of *nitrogen peroxide* as recorded by means of a Jamin interferential refractometer and a small Hilger spectrograph.[1] Both the rapid variations of the refractive index near prominent lines, and the gradual fluctuations of the mean index caused by groups of lines are well shown. Now, from our Fig. 5 (p. 12) it is clear that for a line of given intensity the magnitude of the relative displacement in the spot spectrum will depend on the value which n_0 has in the part of the spectrum under consideration, as well as on the presence of direct neighboring lines. Hence one cannot be astonished at finding very unequal displacements with lines of the same intensity, the same element, and about the same region of the spectrum. This is an important inference, in respect to which our point of view has the advantage over the radial-motion hypothesis.

Unless some other plausible explanation of this peculiar mutual influence of neighboring lines on the magnitude of their displacements be found, we are forced to consider the foregoing results as a direct proof of the efficiency of anomalous dispersion in producing solar phenomena.

Displacements and line intensity.—Our next aim must be to explain the fact that, on the average, the displacements decrease with the increase of line intensity.

This problem brings us in contact with a characteristic feature of our theory that has given rise to some misapprehension and opposition.

Line displacements caused by Doppler effect, Humphreys effect, and Zeeman effect will increase in proportion as the velocity, the pressure, and the strength of the magnetic field increase; but the analogous inference that solar line displacements caused by

[1] *Proc. Roy. Acad. Amsterdam*, **13**, 1088, 1911; *Zeitschrift für wissenschaftliche Photographie*, **10**, 62, 1911.

anomalous dispersion should always increase in proportion as the degree of anomalous dispersion in the sun increases, or that they should even be proportional to the refraction effects observed with the corresponding lines in the laboratory, is entirely erroneous.

This, of course, does not involve the assertion that results on anomalous dispersion obtained from terrestrial sources would be without any value for the interpretation of solar phenomena. On the contrary, in the theory advanced, as in any other interpreting system, observation aided by experimental research is the only reliable basis; but in criticizing the conclusions of the rival theories it should be noted that the relations of anomalous dispersion are very different from those of other causes of line displacements, and require special study.

In order to compare the refraction effects associated with weak and with strong lines we once more refer to the schematic Figs. 1–4 on p. 9, and to their interpretation given in the paper mentioned in the footnote of p. 10. The case represented is an ideal one, not only because we gave the depression a spherical shape, but also on account of the assumed smoothness of the gradients.

We shall now consider the effect of slight irregularities of optical density superposed upon the systematic gradients due to the vortex.

Waves for which $\pm(n-1)$ is not very much greater than n_0-1 (n_0 being the mean refractive index for the spectral region under consideration) will essentially behave in accordance with the ideal case, although their paths will appear somewhat sinuous on account of the small fluctuations of the density. Such are the conditions obtaining with the R-light and V-light of lines of low intensity. The aspect of those lines will therefore nearly correspond to the shape represented in Fig. 4. If a wave-length belonging to the R-light of a weak line could be isolated with the spectroheliograph, the solar image thus obtained would show the spot displaced toward the limb; similarly a wave-length belonging to the V-light would show the spot displaced toward the center of the disk. This involves that in the spectrum of the limb-edge the intensity of the dark line falls off sharply on the violet side, pro-

gressively on the red side, and just the reverse in the spectrum of the center-edge of the spot.

The following weak lines, visible on Fig. 1 of the plate[1] of St. John's first paper, show this characteristic of our schematic line (Fig. 4, p. 9, where the effect is exaggerated) unmistakably: λλ 4750.1, 4751.28, 4764.5, 4764.72, 4768.85, 4776.26, 4778.4, 4781.9.

The measured values of the displacements of lines of this kind are determined by the difference of wave-length between the "centers of gravity" of their R-light and V-light areas, in so far as the width of the true absorption line, common to both edges of the penumbra, may be neglected.

The peculiar shape which these lines of very low intensity show in the spot spectrum when observed with radial slit corroborates the fundamental hypothesis of our solar theory, viz., that the width of Fraunhofer lines is in the main an effect of anomalous dispersion.

Proceeding to the case of a stronger line, we are concerned with waves for which $\pm(n-1)$ has such high values that even the lesser, parasitical density gradients make those rays deviate very sensibly. The rays will then follow winding paths entirely different from the smooth lines of the drawings, Figs. 1 and 2, p. 9. Thus, e.g., a ray of V-light emerging from the peripheral part of the penumbra, which if only moderately refracted would have carried much energy (according to Fig. 2), will now on account of its frequent curving possibly take the energy from a less favorable direction, and will at all events have suffered more loss by scattering, both molecular and refractional, than have waves for which $\pm(n-1)$ is smaller.

We may also consider the matter thus: very strongly refrangible rays are not so much influenced by the large-scale density configuration of the vortex region. In fact, such rays are refracted by the irregular gradients outside as well as inside the vortex regions; and although equal positive and negative values of $n-1$ determine opposite incurvations, the paths are everywhere so twisted throughout the whole layer corresponding to the levels where sun-spots occur, that the combined effects of those waves blend into a

[1] Cf. *Mt. Wilson Contr.*, No. 69, Plate XXIV; *Astrophysical Journal*, **37**, 324, 1913, Plate XII.

vague, fine-grained structure, and the systematic spot-gradients are scarcely indicated by them.[1]

In the spectrum of the spot, therefore, the strongly refracted waves will not in general produce any marked asymmetrical phenomenon. By their winding and scattering amid the lesser density fluctuations they get possibly still more weakened inside than outside the vortex region, and thus make the line appear strengthened and widened,[2] but nearly equally so on both edges of spot and line, the average effect being almost the same for R-light and V-light. Waves a little farther from the core of the line, however, are less refracted and behave according to the scheme that holds good for weak lines; hence the shading of the line will be broader on the red than on the violet side in the peripheral penumbra, and broader on the violet than on the red side in the penumbra directed toward the center of the disk. This makes it appear that the line is shifted bodily. When the relative displacement is being measured, the strong central part which the lines of both spot-edges have in common will preponderate in the determination of the "centers of gravity"; in this way the displacement will come out smaller than with lines of low intensity. This diminution of the distance between the estimated centers, progressive with increasing line intensity, explains the law discovered by St. John.

Displacements and wave-length.—We shall now discuss from our point of view the connection that seems to exist between displacements and wave-length.

Together with the data given in Tables II and III the contents of Table VI, graphically represented in Fig. 6, may serve to provide us with a survey of the available material.

[1] Spectroheliographic images obtained with the very centers of strong lines really do not show the spots (Deslandres); they give, however, some coarse details corresponding to higher levels, where the smoother gradients suffice to impart to those highly refrangible rays the deviations necessary for producing contrasts.

[2] The exceptionally wide lines H, K, H_α, H_β, H_γ, H_δ, and some other winged lines require special treatment, because with them very probably the middle part of the dispersion-curve uniting the minimum with the maximum will have to be taken into consideration. A discussion of the enhanced lines and those weakened in the spot spectrum must also be postponed until the completion of a laboratory investigation now in progress.

In the first column of Table VI are indicated ten regions of the spectrum, including all the observed lines; the second column contains the numbers of the lines measured in each region; in the

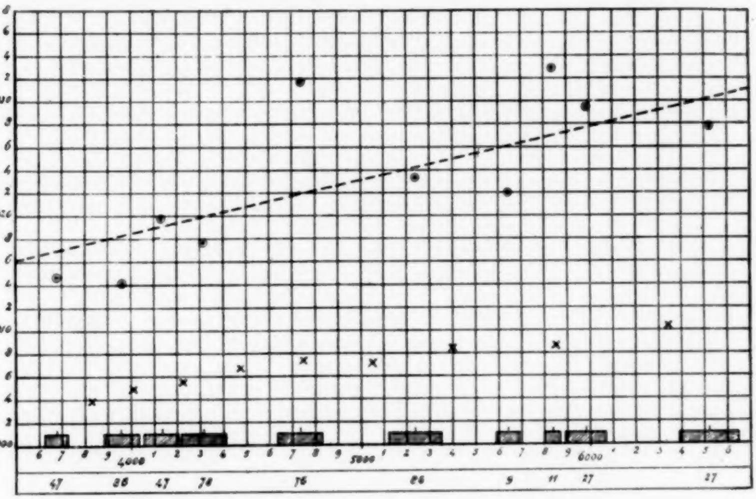


FIG. 6.—Mean displacements in successive regions of the spectrum
⊙ ⊙ ⊙ relative displacements in spot spectra (St. John)
× × × displacements at the sun's limb (Adams)

TABLE VI

DISPLACEMENTS AND WAVE-LENGTH

Region of Spectrum	Number of Lines	Mean λ	Mean Displacement
λ 3625 to 3725	47	3675	0.0147
3880 to 4035	86	3957	.0141
4055 to 4205	47	4130	.0197
4215 to 4410	78	4312	.0176
4635 to 4830	76	4732	.0317
5120 to 5350	86	5235	.0233
5590 to 5690	9	5640	.0219
5800 to 5870	11	5835	.0328
5890 to 6070	27	5980	.0294
6390 to 6650	27	6520	0.0277

third and the fourth columns are the mean wave-length and the mean displacement for each region.

Although on the average the displacements obviously increase with wave-length, the most striking feature of the table and the

figure[1] is the great variety of the *mean* values along the spectrum. One cannot admit such fluctuations to be entirely accidental or due to observational errors. The important retrograde difference between the values for mean λ 4732 and mean λ 5235, e.g., far exceeds the mean error of the result of 76 and 86 observations, and must be genuine.[2] If the Doppler effect were the cause of the displacements, their variation with wave-length would be represented by the straight line shown in the figure. The great deviations from this line prevent us from considering the data as decisively in favor of the hypothesis that we are dealing with a radial motion phenomenon. We suggest under reserve the following explanation, in which the combined effects of scattering and refraction are considered.

Suppose for a moment that the irregular density gradients of the solar gases had all disappeared, only the slow radial gradient being left. We should then look down through a perfectly calm, moderately transparent medium upon an evenly luminous background. The brilliant core of the sun would be seen through a kind of haze or fog caused by molecular scattering. As the scattering coefficient varies inversely as the fourth power of the wave-length, any definite degree of fogginess would be found for violet light at a higher level than for red light. If there were self-luminous or absorbing objects in the medium, they would be visible down to greater depths with red waves than with violet waves.

Now let the density gradients reappear, and with them the sinuating of rays and the resulting irregular distribution of the light. Evidently the beams of red waves will have traveled longer distances through the sun before emerging than the beams of violet waves; at the lower levels the average density gradients

[1] The mean relative displacements in the spot spectrum are indicated by small circles. The crosses in the lower part of the figure give the mean values of the displacements of Fraunhofer lines *at the limb* for successive regions of the spectrum, as deduced from measurements made by Adams.

[2] It is worth noticing that this anomaly occurs at the same place in the spectrum where the curve representing the means of Adams' measurements of limb displacements as a function of wave-length also sinuates (cf. Fig. 6), and where H. C. Vogel's well known table of spectrophotometric observations on the distribution of various kinds of light on the solar disk shows a similar anomaly. We do not venture to explain these remarkable coincidences as yet, although the anomalous-dispersion theory indicates connective points.

probably are steeper; red light therefore has in general more opportunities to get strongly deviated, and thus to produce contrasts, than violet light. This may account for the general tendency of the displacements to increase with increasing wave-length.

As to the fluctuations of the mean displacements along the spectrum, it seems possible to make the oscillating values of n_0-1 responsible for them, in the manner already discussed on p. 19.

ON ST. JOHN'S APPRECIATION OF ANOMALOUS DISPERSION AS A POSSIBLE CAUSE OF THE RELATIVE DISPLACEMENTS

Although the preceding pages implicitly contain a reply to the greater part of St. John's critical remarks on anomalous dispersion, it would seem proper still to discuss briefly his principal objections seriatim.

In judging of the degree of correspondence between the figure illustrating the way in which anomalous dispersion acts in producing the shifts (Fig. 4, p. 9) and the aspect of the lines found on the plates, St. John confined his attention to the stronger lines. Our schematic figure, however, is not directly applicable to strong lines for the reasons amply discussed on p. 21. In the case of weak lines the correspondence appears to be quite satisfactory.

Of the three facts mentioned by St. John as requiring explanation from the point of view of anomalous dispersion, two have been explained in this paper, viz., the variation of displacement with wave-length, and the variation with the intensity of the lines. The fact that a few lines (corresponding to prominent chromospheric lines) are displaced in the opposite direction has provisionally been left out of consideration in view of a special research now in progress. On the other hand, a fourth and very marked fact not explained by St. John's theory, viz., the large deviations of individual displacements from the means for each intensity class and spectral region, is shown to be in harmony with our interpretation on the basis of anomalous dispersion.

In a note on the interpretation of spectroheliograph results and of line-shifts[1] I explained why Mr. Adams, when comparing

[1] *Astrophysical Journal*, 31, 428, 1910.

certain laboratory results on anomalous dispersion with the displacements of Fraunhofer lines at the limb,[1] failed to find any clear relationship between the two phenomena. Mr. St. John criticizes this explanation. He quotes from my paper:

> That a simple comparison of Geisler's observations on anomalous dispersion of metallic vapors in the arc with displacements at the limb—as given by Adams on page 28—could not possibly serve the purpose of finding such a relationship is evident. . . . .

Now, this was only a fraction of a sentence; the continuation of it runs thus:

> for the amount of that part of the displacement which is due to anomalous dispersion is determined by the degree of asymmetry of the Fraunhofer line under consideration; and this asymmetry is not a mere property of the corresponding element itself, revealable in laboratory experiments, but depends on the concentration with which that element is represented in the solar atmosphere.

But instead of completing the quotation by adding these explanatory lines, St. John comments as follows:[2]

> In view of the consideration that the basis of all astrophysical investigations rests upon the fundamental postulate that direct comparison is possible between the spectrum results obtained from terrestrial sources and the behavior of the spectrum lines in solar and stellar spectra, the first statement in the quotation is somewhat remarkable.

I am sure Mr. St. John would not have suggested to his readers such a bad opinion of my working method if he had realized the meaning of the part of my argument which he represented by an ellipsis. Indeed, from the point of view that Fraunhofer lines are dispersion bands, their asymmetry is due to the fact that, for the narrow region of wave-length surrounding each separate line, $n_0 - 1$ generally differs from zero (n_0 being determined by the composition of the solar gaseous mixture). The degree of asymmetry depends on both n_0 and the anomaly produced by the line itself.[3] As in Geisler's experiments the solar values of n_0 did not enter, nor anything analogous to them, the magnitude of the displacements of

[1] *Astrophysical Journal*, **31**, 57, 1910.

[2] *Mt. Wilson Contr.*, No. 74, p. 43.

[3] *Proc. Roy. Acad. Amsterdam*, **12**, 281, 1909; *Physikalische Zeitschrift*, **11**, 68, 1910.

the Fraunhofer lines at the limb bears no relation at all to the results of those experiments. This assertion does not imply any disregard of the necessity of testing theories by experimental research wherever possible, provided that the test be based on sound reasoning.

St. John's next criticism bears on the following statement which he quotes from the same paper: "A peculiar feature of our explanation is that both very strong and very weak anomalous dispersion make the displacements small, whereas intermediate values give larger displacements."

We were here concerned with displacements at the limb. Relative displacements at opposite edges of the spot spectrum have an entirely different origin. It is clear, indeed, that if a Fraunhofer line happened to be perfectly symmetrical ($n_0 - 1 = 0$) in the spectrum of the mean photospheric light, anomalous dispersion would not displace it at the limb, but would nevertheless produce a relative displacement at the opposite edges of the spot spectrum. The sign of $n-1$ is material to the Evershed effect, but almost immaterial to the Adams effect.

This fundamental difference between the two kinds of solar displacements escaped St. John's notice. In Table XXII (*Contribution No. 74*, p. 43) he uses some of his observations on spot lines in order to disprove my contention regarding limb lines! The same confusion runs through pp. 44 and 45 of the paper; this is the cause of St. John's finding so many discrepancies between the results of his observations and what he wrongly believes to be requirements of the dispersion theory.

I can easily show, using Adams' measurements of the displacements of the Fraunhofer lines at the sun's limb,[1] that the above-quoted deduction from the anomalous-dispersion theory is in perfect harmony with the facts.

Adams himself considers pressure as the effective agent in producing these displacements; it therefore did not occur to him to classify the shifts according to line intensity. Now accomplishing this classification, we obtain the synopsis given in Table VII.

[1] Adams, *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

The result is very striking; the shifts are greatest for lines of intensity 5, 6, and 7, and decrease progressively for the lower as well as for the higher intensities. This is exactly what the theory requires on the view that the intensity of Fraunhofer lines is chiefly determined by anomalous dispersion, and that their apparent displacements toward the red are simply due to the inequality of the average refraction suffered by the R-light and the V-light of

TABLE VII

SYNOPSIS OF ADAMS' MEASUREMENTS OF THE DISPLACEMENTS OF LINES AT THE LIMB, SHOWING THE MEANS FOR EACH LINE INTENSITY

	Intensity										
	0	1	2	3	4	5	6	7	8	9–12	15–40
Number of lines measured.......	7	51	99	106	71	40	41	14	12	11	15
Mean displacement (unit=0.001 A)	3.6	5.5	6.6	6.8	7.1	8.8	8.3	8.8	7.9	5.3	3.0

TABLE VIII

SYNOPSIS OF ADAMS' MEASUREMENTS OF THE DISPLACEMENTS OF LINES AT THE LIMB, SHOWING THE MEANS FOR NINE CONSECUTIVE REGIONS OF THE SPECTRUM

	Region of Spectrum								
	3740–3923	3923–4100	4100–4350	4350–4600	4600–4900	4900–5200	5200–5600	5600–6100	6100–6580
Number of lines.....	52	54	70	76	38	41	54	34	50
Mean displacement (unit=0.001 A)...	3.9	4.9	5.5	6.7	7.4	7.1	8.4	8.6	10.3

each line. Indeed, the apparent displacement must then always be a *fraction* of the width of the line; it therefore decreases with line intensity. The fraction, however, will in general be smaller with wide lines than with narrow lines (for it depends on the variable proportion between $n-1$ and n_0-1), thus making the asymmetry less conspicuous in the case of the wide lines. Lines of moderate intensity therefore show the largest mean displacements. And because the value of n_0-1 fluctuates along the spectrum,

especially in the vicinity of strong lines, we also conceive that in every intensity class the individual displacements may deviate widely from the mean, as they really do.

Table VIII contains the means of Adams' measurements for nine consecutive regions of the spectrum. They are plotted in the lower part of Fig. 6, and give evidence of a progressive increase with wave-length, excepting the anomaly between λ 4500 and λ 5500 already alluded to in the footnote on p. 24. This variation with the wave-length,[1] exhibited by the general shifts of the Fraunhofer lines toward the red, may perhaps be explained on the same basis as the corresponding variation observed in the case of the relative displacements in the spot spectrum (cf. p. 24). Both phenomena would seem to be due to the united influence of refraction and molecular scattering, and the refraction effects would be greater for the longer waves.[2]

Continuing the discussion of St. John's criticism, we arrive at his statement that relative displacements are sometimes observed in the spot spectrum when the slit of the spectrograph is perpendicular to the radius of the solar disk passing through the center of the umbra. It is argued that such displacements are very simply explained as Doppler effects, indicating occasional cyclonic movement, and that, on the other hand, the dispersion theory is unable to account for them. The latter inference, however, supposes the region of minimum density to have an ideally symmetrical shape. As deviations from that condition are quite probable, there is no difficulty in accounting for the occasional displacements

[1] In a paper, "Les Raies de Fraunhofer et la dispersion anomale de la lumière," published in *Le Radium*, **7**, October 1910, I suggested that the variation with wave-length here considered might be due to a general increase of n_0 with the wave-length, but I am now inclined to think that the influence of refractional scattering is more effective.

[2] A general displacement of the Fraunhofer lines toward the red, proportional to the wave-length, and amounting to about 0.010 A for λ 5000, is required by the gravitation theories of Einstein (*Annalen der Physik*, **35**, 905, 1911) and Nordström (*ibid.*, **42**, 549, 1913). Gravitation may thus contribute to the production of the observed shifts, but it certainly is not their main cause, since it does not account for the principal features of the phenomenon: the great variability of the shifts from line to line, and the marked relation between the mean shifts and the intensities of the lines.

in question on the basis of unequal refraction at opposite edges of the spot.

In the case of the *winged lines*, I had originally assumed that the core of the line was a pure absorption effect. In later publications,[1] evidently not considered by St. John in connection with the present subject, I was led to the conclusion that even the cores of those winged lines might be influenced by anomalous refraction and scattering. Probably a thorough treatment of these cases will prove to be difficult because the electronic theory of the dispersion, scattering, and absorption of light seems to require some extension in order to make it applicable to the very centers of wide lines. The subject is reserved for further investigation.

St. John's final remarks on the general question how far it seems probable that refraction and anomalous dispersion would produce any solar phenomena may be passed over, because they essentially refer to a paper, "On the Application of the Laws of Refraction in Interpreting Solar Phenomena," by Mr. Anderson.[2] At the time Mr. St. John wrote his criticism, my refutation[3] of Anderson's argument had not yet been published.

SUMMARY

1. The best established general result deduced by St. John from his measurements of the Evershed effect is that the displacements appear to vary progressively with the intensity of the lines. The regular progression of the mean values calculated for successive intensity classes pleads in favor of the general accuracy of the measurements.

2. Means taken for successive regions of the spectrum (Table VI, Fig. 6), though roughly indicating increase with wave-length, run very irregularly. Their deviations from a line representing proportionality with wave-length are too great to be attributable to accidental errors, and therefore prevent us from considering the results as decisively in favor of the hypothesis that the displacements are due to the Doppler effect.

[1] *Proc. Roy. Acad. Amsterdam*, **13**, 881 and 1263, 1911; *Physikalische Zeitschrift*, **12**, 329 and 674, 1911.

[2] *Astrophysical Journal*, **31**, 166, 1910. [3] *Ibid.*, **38**, 129, 1913.

3. St. John's hypothesis of the "Iron Scale," according to which the lines of an element are supposed to originate at a lower level as their intensity is smaller, meets with difficulties from the point of view of the physicist.

4. The insufficiency of indications of vertical motion in sunspots is unfavorable to the hypothesis that the displacements considered are due to radial outflow of matter from spots.

5. A characteristic feature of the displacements is their great diversity of magnitude along the spectrum, even if lines of about equal intensity are considered. This peculiarity, which seems to be inexplicable on the basis of the radial-motion hypothesis, follows immediately from the anomalous-dispersion theory, because from that point of view the displacement of a line in the spot spectrum depends on (*a*) the anomaly of the dispersion-curve produced by the line considered, and (*b*) the value of n_0 which is determined by the other lines, and therefore fluctuates along the spectrum.

6. For each line intensity and spectral region a "normal displacement" can be deduced from St. John's measurements. The dispersion theory requires that the amount of the displacement of a line A will be sensibly influenced by a strong neighboring line B. On the assumption that n_0 is > 1, the influence must be such that if B lies on the red side of A, it reduces the displacement of A as compared with the normal value; if B is situated on the violet side, it must have the opposite effect, but to a lesser degree. This inference is perfectly borne out by all the evidence that can be gathered from St. John's Table I. In so far as other theories appear unable to account for this mutual influence of Fraunhofer lines, we may consider the phenomenon as directly proving the efficacy of anomalous dispersion in the sun.

7. The law connecting the Evershed effect with the intensity of the lines is in harmony with the deductions from the dispersion theory.

8. Judging from the behavior of the weakest lines (for which the optical effect of the general spot gradients is not much disturbed by the effect of superposed irregular density gradients), one gets the impression that nearly the whole width of the Fraun-

hofer lines must be due to anomalous dispersion, or that Fraunhofer lines are in the main *dispersion bands*.

9. The apparently intricate connection between displacements and wave-length seems to be explicable if we consider (*a*) that on account of molecular scattering short waves have on the average less opportunity of being refracted than long waves, and (*b*) that the value of n_0 fluctuates along the spectrum.

10. A discussion of St. John's remarks on anomalous dispersion made it necessary to expatiate on the difference in character which from the point of view of the anomalous-dispersion theory exists between the displacements of spot lines (Evershed effect), and the displacements of lines at the limb, as studied by Adams.

11. If the displacements at the limb, measured by Adams, are classified according to line intensity, and averaged, the means are found greatest for intensities 5, 6, and 7, and gradually decrease for greater as well as for smaller intensities. This law was predicted a few years ago by our theory; it will be difficult to explain it on the basis of the current interpretation of those displacements as a pressure effect.

Physical Laboratory
University of Utrecht
March 1914

ON BRIGHTNESS AND CONTRAST IN OPTICAL IMAGES

By P. G. NUTTING

The brightness of an image has been treated theoretically by a number of investigators and quantitative measurements have been attempted by several. However, both the theory and its experimental verification appeared so unsatisfactory that it seemed desirable to obtain a more nearly complete solution of the theoretical problem and to develop methods for making more precise direct measurements of relative illumination.

The theory here given is believed to be complete for portions of object and image near the axis and normal to it, and for an object at any distance with any focal length of lens or mirror. It has not been extended to oblique pencils. On the other hand, experimental methods have been developed to a point where determinations are scarcely more uncertain than photometric settings. The experimental method finally adopted was, in brief, to form an image of an extended plane source, by means of the lens under test, upon a matte, white-reflecting surface of magnesium carbonate. The illumination of this image was compared with that of the same spot with the lens removed.

Precision is attained by (*a*) using light from the *same* source to illuminate both lens and comparison surface of the photometer, thus avoiding errors due to fluctuation in the source, and (*b*) by careful determination of stray light. Calling I_0 the illumination at the source (strictly the normal luminous flux), I_1 that at the receiving screen without the lens, and I_2 that with the lens, the ratio I_1/I_0 is computed from dimensions and distances, while I_1/I_2 is observed with the photometer, hence I_2/I_0, the relative illumination of object and image, is obtained.

The actual source used was a 1500 candle-power tungsten lamp in a cubical white-lined box, the front face of which was a sheet of opal glass. This face was 33.5×35.5 cm and in use emitted about 0.7 candle per sq. cm. At about 5 meters from this face was the receiving block. The photometer used was a modified Beckstein

illuminometer. This was sighted directly on the image spot on the receiving screen, after removal of the receiving screen attached to the instrument. The comparison screen of this instrument is ordinarily illuminated by a glow lamp contained in a white-lined sphere; this was removed and light directly from the testing source

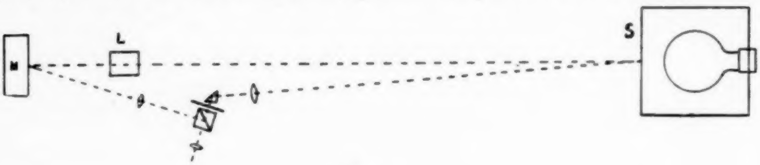


FIG. 1

reflected on the screen, so that fluctuations in the source were of no consequence. The final illumination in the photometer cube was ample for maximum sensibility.

In previous treatments of the theory of image illumination there appears to have been no complete consideration of the foreshortening of zone pencils. The error is not negligible with aperture ratios greater than F/8, and in modern high-speed objectives becomes quite large. Further difficulties have arisen from confusion of terms relating to luminous intensities.

Consider an axial element ds_0 of a plane object normal to the axis. Let the brightness of this object and its diffuseness of emission be such that in the direction of the axis the illumination is dL lumens per unit area of object. Call this intensity I_0. The illumination in the direction of the lens is then such as would be given by a point source of intensity I_0dS_0 at the axial point of the

FIG. 2

object. The axial illumination (flux density) at a distance u from the object is then $I_0\ dS/u^2$. The total flux dL on a normal axial element dS at a distance u is I_0dS_0dS/u^2. The flux from an element dS_0 to an element dS_1 not parallel or coaxial with it, is

$$dL=\frac{I_0dS_0 \cos \alpha \cos \alpha' \ dS_1}{u^2+r^2} \tag{1}$$

where α and α' are the angles between the normals to dS_0 and dS_1 and the line joining them.

In the case of a lens, the element dS_1 is a ring-shaped element passing through the lens, normal to its axis at the intersection of the zonal surface with the axis. The zonal surface is the locus of intersection of corresponding image and object zone pencils. The element dS_1 is strictly an element of the zonal surface, but if this is used the light-integral assumes an exceedingly complex, non-integrable form. The assumption that the zonal surface is plane introduces a second order error which will be discussed and evaluated below.

We have then $dS_1 = 2\pi r dr$ and $\alpha' = \alpha$. On the image side of the lens the light within each cone element from dS_0 falling upon the image element dS_1 is constant and equal to that falling on the ring element dS_1 of the zonal surface, corrected twice for foreshortening by the factor $\cos^2 V$. The total flux on dS is then

$$L = I_0 dS_0 \int_0^R \frac{\cos^2 U \cos^2 V}{u^2 + r^2} dS_1 \tag{2}$$

But,

$$\cos^2 U = u^2/(u^2+r^2) \text{ and } \cos^2 V = v^2/(v^2+r^2)\,,$$

hence

$$L = 2\pi I_0 dS_0 \int_0^R \frac{u^2 v^2 r dr}{(u^2+r^2)^2 (v^2+r^2)} \tag{3}$$

The flux density at the image is L/dS, but $dS_0/dS = u^2/v^2$, hence

$$I = I_0 \pi \left[\frac{u^4}{(u^2-v^2)^2} \log \frac{\dfrac{u^2}{u^2+R^2}}{\dfrac{v^2}{v^2+R^2}} - \frac{u^2}{u^2-v^2} \frac{R^2}{u^2+R^2} \right] \tag{4}$$

This is the complete expression for relative illumination I/I_0 of object and image, for any object distance u, image distance v, and any aperture $2R$.

The correction for curvature of zonal surface may be found as follows, making use of the sine condition. The equation of the zonal surface is

$$\frac{\sin V}{\sin U} = \text{const.} = \frac{u}{v}$$

or in Cartesian co-ordinates (see Fig. 3)

$$\frac{(u+x)^2+y^2}{(v-x)^2+y^2}=\frac{u^2}{v^2}$$

$$(u^2-v^2)y^2=v^2(u+x)^2-u^2(v-x)^2$$

Hence for any y the zonal surface makes an angle with the axis whose tangent is

$$\frac{dy}{dx}=\frac{v^2(u+x)-u^2(v-x)}{y}$$

The most unfavorable case is evidently for $U=0$, $u=\infty$. In this case

$$\frac{dy}{dx}=\frac{v-x}{y}$$

or the zonal surface is a circle with v as a radius and the image as center.

Now the error in (2) due to assuming the zonal surface plane is an error in the product $P=\cos U \cos V$, in which U is increased and

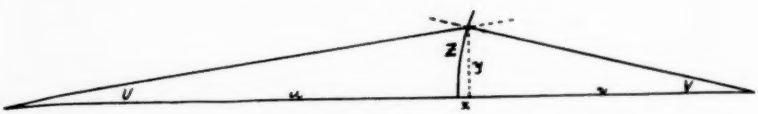


FIG. 3

V decreased by the same small angle. Differentiating this product and putting $dV=-dU$, the error in P is $dP=-\sin(U-V)dU$. In the least favorable case ($U=0$) the maximum error is less than 0.005 in the case of an $F/8$ lens, or 0.02 for an $F/3.5$ lens.

The general expression for relative illumination (4) may be expressed in the simpler form:

$$\frac{\rho}{\pi}=\frac{1}{(1-m^2)^2}\log\frac{\cos^2 U}{\cos^2 V}-\frac{\sin^2 U}{1-m^2} \qquad (5)$$

by putting $\rho=I/I_0$ and $m=v/u$ and substituting $\cos U$, $\cos V$, and $\sin U$ for their values in u, v, and R.

The form best adapted for computation is

$$\frac{\rho}{\pi}=\frac{1}{(1-m^2)^2}\log\frac{1+\left(\dfrac{a}{1+m}\right)^2}{1+\left(\dfrac{am}{1+m}\right)^2}-\frac{1}{1-m^2}\frac{1}{\left(\dfrac{1+m}{am}\right)^2+1}\tag{6}$$

$$=M^2\log\frac{1+S}{1+T}-M\frac{I}{1+T}$$

since the quantities given are m and aperture ratio of lens, $F/2R=1/2a$.

In the special case of object and image at equal distances ($u=v=2F$) from the zonal surface, (4), (5), and (6) are indeterminate but (3) integrates into

$$\frac{\rho}{\pi}=\frac{R^2(2u^2+R^2)}{(u^2+R^2)^2}=\frac{2w^2+1}{2(w^2+1)^2},\ w=\frac{R}{u}=\frac{2}{a}\tag{7}$$

The special case of an infinitely distant object is of frequent occurrence. In this case

$$\rho=-\pi\log\cos^2 V=\pi\log(1+a^2)\tag{8}$$

which, for V small, approximates closely to

$$\rho=\pi\sin^2 V=\pi a^2\tag{9}$$

the form usually quoted. This is the relative illumination that would obtain if a flux density equal to that at the object came from the zonal surface itself.

In Table I are given computed ratios of flux density at image to that at the object for various apertures and various ratios of image

TABLE I

Aperture	$m=0$	$m=0.1$	$m=0.2$	$m=0.5$	$m=1$	$m=2$
1	0.704	0.580	0.521	0.333	0.179	0.0775
2	.1902	.1580	.1321	.0864	.0491	.0210
5	.0312	.0255	.0219	.01296	.00785	.00347
10	.00785	.00625	.00553	.00349	.00197	.000870
20	.00196	.00162	.00134	.000878	.000490	.000217
50	.000314	.000259	.000216	.000135	.000078	.0000346
100	0.000078	0.000065	0.000051	0.000044	0.000019	0.0000086

distance to object distance ($v/u=m$), neglecting losses by reflection and absorption.

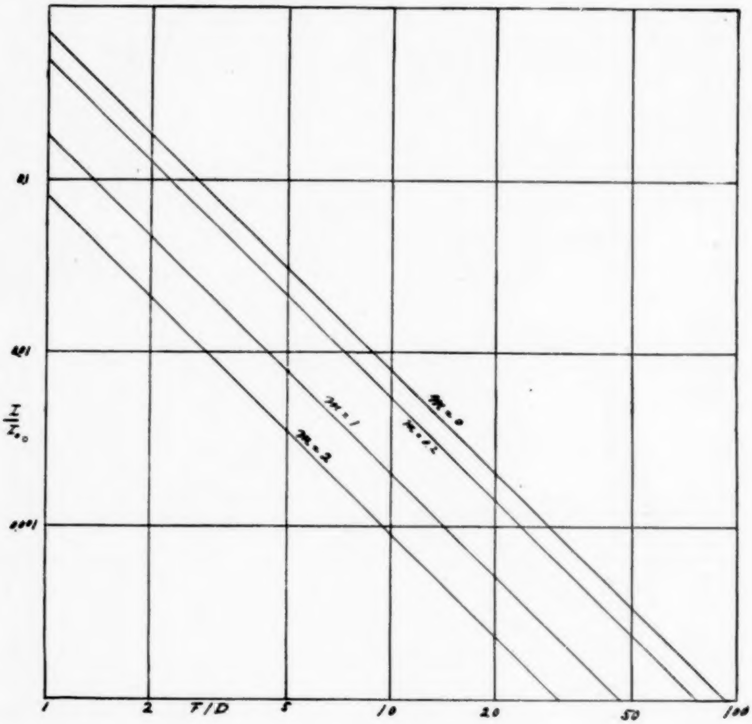


FIG. 4

The experimental data taken are given in the following tables and figures. They refer to the lenses listed below:

1. Cooke Process, 18″, Series V, No. 32051, tested at F/8 to F/90. E.F.L. 46.7 cm, $I_1/I_0=0.00475$, $m=0.08654$.
2. Cooke Process, 11″, Series V, No. 31796, tested at F/8 to F/32. E.F.L. 28.6 cm, $I_1/I_0=0.00475$, $m=0.0675$.
3. Cooke Process, 7½″, Series V, No. 33991, tested at F/8 to F/32. E.F.L. 19.5 cm, $I_1/I_0=0.00475$, $m=0.0417$.
4. B. & L.-Zeiss Tessar, Series IC, No. 1320682, tested at F/5.4 to F/32. E.F.L. 15.05 cm, $I_2/I_0=0.00494$, $m=0.0309$.
5. Cooke, Series II, F/4.5, No. 40399, tested at F/4.72 to F/22. E.F.L. 20.93 cm, $I_1/I_0=0.00499$, $m=0.046$.

6. Fuess telescope objective (cemented doublet), F/5.3
   E.F.L. = 15.3 cm, Ap. 2.90 cm, $I_1/I_0 = 0.00494$, $m = 0.033$.
7. B. & L.-Zeiss Tessar, Series IC, No. 95933, 40 mm, F/4.5, tested at F/4.94.
8. Cooke Cinematograph, 2″, F/3.5, tested at F/3.67.
9. Zeiss-Krauss Cinematograph Tessar, F/3.5, 75 mm (77.0/19.5).
10. Zeiss-Krauss Cinematograph Tessar, F/4.5, 150 mm (152.1/32.3).

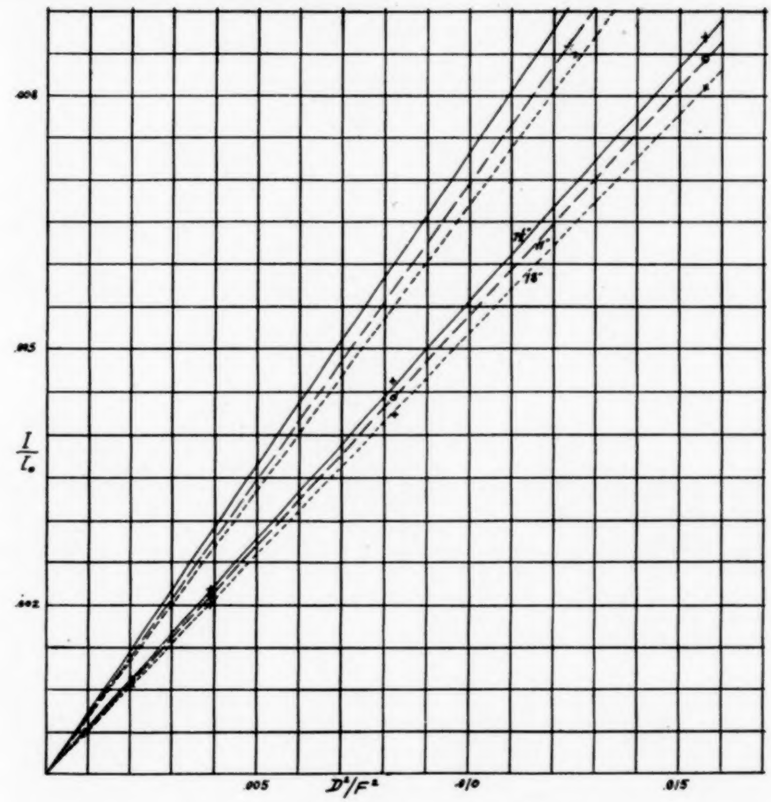


FIG. 5.—Tests of three process lenses

All apertures and focal lengths were determined on a precision lens bench; the marked stop points were correct to within the error in setting, except the lower marks on the high-speed lenses (F/4.5 and F/3.5). Uncertainty in setting the stop at a given mark (in measuring illumination) amounted to roughly 2 or 3 per cent, hence the results are uncertain to about 5 per cent. The

slope of the line relating I_2/I_0 to R^2/F^2, used in obtaining the specific transmission, is uncertain to certainly less than 3 per cent. The three process lenses show a sensibly linear relation between I_2/I_0 and D^2/F^2 $(D=2R)$. The observed brightness divided by the computed brightness is taken as a measure of the percentage transmission. The constancy of this for different stop openings indicates the precision of setting and measurement.

TABLE II

Stop	18″ Cooke, Ser. V			11″ Cooke, Ser. V			7½″ Cooke, Ser. V		
	Obs.	Comp.	Trans.	Obs.	Comp.	Trans.	Obs.	Comp.	Trans.
F/8	0.00820	0.01058	0.774	0.00841	0.01085	0.776	0.00870	0.01136	0.766
11	.00374	.00459	.816	.00441	.00570	.774	.00463	.00600	.770
16	.00206	.00264	.783	.00210	.00270	.779	.00202	.00283	.714
22	.00108	.00140	.773	.00116	.00143	.781	.00115	.00150	.766
32	.000524	.000661	.792	0.00051	0.00067	0.762	0.00050	0.00071	0.72
45	.000268	.000336	.796	......	......	....	......	......	....
64	.000156	.000166	.94	......	......	....	......	......	....
90	0.000100	0.000083	0.83	......	......	....	......	......	....

There are six free surfaces in these lenses. The sixth root of 0.77 is 0.956, a percentage transmission per surface corresponding to a refractive index of 1.53. Hence the observed brightness of image is slightly (about 4 per cent) higher than would be expected.

The Tessar and Cooke F/4.5 lenses gave results shown in Table III.

TABLE III

B. & L.-Z. Tessar IC				8″ Cooke, Series II			
Stop	Obs.	Comp.	Trans.	Stop	Obs.	Comp.	Trans.
F/4.74	0.0249	0.0328	0.758	F/4.72	0.0237	0.0321	0.739
5.66	.0184	.0238	.724	5.68	.0172	.0232	.744
8	.00890	.0115	.766	8	.00836	.0112	.749
11	.00476	.00610	.765	11	.00425	.00595	.715
16	.00231	.00288	.804	16	.00210	.00282	.664
22	.00115	.00152	.765	22	0.00117	0.00149	0.800
32	0.00054	0.00072	0.750	......	......	......	......

The remaining small lenses (m sensibly zero) gave:

Fuess telescope objective, cemented doublet, F/5.32: $a=0.0940$, observed I_2/I_0 0.0257, computed 0.0278, ratio 0.921.

B. & L.-Zeiss Tessar, IC, F/4.5, 40 mm: F/4.94, $a=0.1012$, observed $I_2/I_0$0.0172, computed 0.0322, ratio 0.535.

2″ Cooke Cine, F/3.4, tested F/3.67: $a=0.1360$, observed $I_2/I_0$0.0393, computed 0.0585, ratio 0.673.

Zeiss-Krauss Cine Tessar, F/3.5, 75 mm, tested at F/3.69: $a=0.1355$, observed $I_2/I_0=0.0330$, computed 0.0579, ratio 0.571.

Zeiss-Krauss Cine Tessar, F/4.5, tested at F/4.71: $a=0.1063$, observed $I_2/I_0=0.0234$, computed 0.0355, ratio 0.660.

CONTRAST IN OPTICAL IMAGES

Large details of an image are of course of the same *relative* brightness as the corresponding details of the object. But details beyond the resolving power of the instrument may gain or lose enormously in contrast relative to their background, depending upon whether these details be points or lines darker or brighter than the background. Such observations as those on the minor satellites and planetary markings depend largely upon the choice of instruments.

For a portion of a telescopic image well above the limit of resolution, the relative brightness of image and object[1] is proportional to the square of the ratio of diameter of objective to its focal length (D^2/F^2). This applies to images formed on photographic plates or other focusing screens. The brightness of an image viewed by the eye is independent of aperture and focal length, I/I_0=constant. Details below the limit of resolution are spread out over a width (δ) proportional and approximately equal to $\lambda F/D$.

If the unresolved detail is the image of a point, its brightness is proportional to relative area of objective and diffraction disk or to D^2/δ^2, if the image of a fine line to D^2/δ. We have then for relative illumination of image and object, quantities proportional in these cases to a constant for a visual image; for an image on a plate or screen to:

$$D^2/F^2=A^2 \text{ for plain object}$$
$$D^2/\delta^2=A^2D^2/\lambda^2 \text{ for point object}$$
$$D^2/\delta=AD^2/\lambda \text{ for line object}$$

From these relations the conditions for maximum contrast in the image are readily determined.

[1] See Nutting, *Applied Optics*, p. 79.

For *light* details[1] on a darker background, maximum contrast in a photographic image is obtained by increasing the brightness of detail over that of the background. If the object is a point, there is a clear gain in increasing diameter of objective D, keeping aperture ratio A constant. On the other hand, there is no gain by increasing relative aperture A by decreasing focal length F, keeping diameter D of objective constant.

For a bright-line object on a darker ground, relative contrast of image and object is proportional to DF/λ. This is enhanced by increasing either D or F or both. Increasing relative aperture by increasing D and decreasing F in the same proportion does not affect relative contrast. In a *visual* image, contrast of bright point details on a darker ground is proportional to A^2D^2, of line details to AD^2, hence there is a clear gain in increasing either aperture ratio or diameter of objective.

With *dark* details on a lighter background the conditions for maximum contrast are quite different. If the dark detail is a point, photographic contrast is proportional to $1/D^2$, and diameter of objective D should be small, aperture ratio A being of no consequence. If the detail is a dark line, then both D and F should be small to give best results.

In observing or photographing fine, dark details on Mars, for example, in so far as contrast is concerned, there is practically no advantage but a considerable disadvantage, aside from atmospheric disturbances, in using a large telescope. W. H. Pickering (4th Monthly Report on Mars) prefers in practice about an 8-inch objective as the best compromise.

Contrasts in dark details on a lighter ground in *visual* images are proportional to $1/A^2D^2$ and $1/AD^2$ for points and lines respectively; hence, in this case also, objectives of lesser aperture and focal length give better contrast.

While it may not be practicable to make use of these principles in the construction of a telescope for general purposes, they are of interest in explaining the widely varying images of the same object obtained with different instruments.

EASTMAN RESEARCH LABORATORY
ROCHESTER, N. Y.
January, 1914

[1] Wadsworth, *Astrophysical Journal* **6**, 119–35, 1897.

ON THE INDIVIDUAL PARALLAXES OF THE BRIGHTER GALACTIC HELIUM STARS IN THE SOUTHERN HEMISPHERE, TOGETHER WITH CONSIDERATIONS ON THE PARALLAX OF STARS IN GENERAL[1]

By J. C. KAPTEYN[2]

I. INTRODUCTION AND SUMMARY

In this paper a first attempt is made to find the parallax of practically all the helium stars brighter than the sixth magnitude for that part of the sky which lies between galactic latitudes $\pm 30°$ and galactic longitudes 216°–360°. I hope in a subsequent paper or papers to deal with the helium stars in the other parts of the sky. For the brighter stars of other spectral classes I have not here tried to derive individual parallaxes, but have discussed somewhat at length the prospects for the successful treatment of such an investigation. A few years ago the undertaking might well have seemed hopeless. Since the discovery of the phenomenon of "star-streaming," however, the outlook has become so much brighter that it seems necessary to look somewhat more closely into the matter. Finally, I have devoted a few pages to the consideration of what we may reasonably hope to achieve for the fainter stars.

At the fourth conference of the Solar Union in 1910, I made some preliminary statements on the results of a study of the helium stars.[3] Attention was drawn to the extraordinary parallelism shown by the proper motions of these, and at least a certain number of the early A stars, in some parts of the sky. Shortly afterward it appeared that for the region of the constellations of Scorpius and Centaurus the phenomenon had also been noticed independently by other astronomers. Opinion, however, was divided on the question whether or not the stars in this particular region of the sky form a separate group. While Eddington and I held that they

[1] *Mt. Wilson Contr.*, No. 82.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[3] *Trans. Internat. Solar Union*, 3, 215–231, 1911.

do, this view was contested, even before my paper had appeared in print, by Campbell and B. Boss.

I hope that the present paper will sufficiently indicate the real state of affairs. Nevertheless, the question is, in my opinion, for the present of only secondary importance. The question of real importance is: Is the parallelism and equality of motion in this part of the sky of such a nature that we can derive individual parallaxes? Considerations as to whether or not this motion is almost wholly due to the progressive motion of the solar system, and as to whether the stars in other parts of the sky participate in it, may be interesting in themselves but do not touch the main conclusion either of my paper in 1910 or of the present one. This is the reason why I shall not reply expressly to the various objections raised.

As to the main point—the determination of individual parallaxes—though its possibility was pointed out in 1910, it could not then be carried out because sufficient radial-velocity data did not then exist. For the purpose of supplying such data, and more generally in order to provide a solid basis for a complete study of the B and early A stars, the Mount Wilson Solar Observatory resolved to place a large number of these stars on the observing program. Personally I tried to derive, and gradually to improve, whatever results could be obtained from the still incomplete data, in order to determine what further observations were most desirable. These observations are now approaching their termination and it will thus be possible very soon to bring the investigation of the helium stars, and of part of the A stars, to a provisional conclusion. In the meanwhile Campbell's catalogues of radial velocities of the B and A stars have appeared. For the northern sky the Lick measures, as well as those of the Yerkes, Allegheny, and other observatories, supplement the data collected at the Mount Wilson Solar Observatory in the most desirable way. For the southern sky Campbell's catalogues and the published contributions of the Cape Observatory constitute practically the whole of the available material.

The fact that the Mount Wilson observations are not fully completed, whereas for the southern sky we have definitive values which, for some time probably, will not be very greatly extended,

has been one of the reasons for beginning a somewhat more complete study of the helium stars with the Southern Hemisphere. Another and even more important reason for this choice lies in the far greater regularity shown by the astronomical proper motions in this part of the sky. It seems rational to begin the investigation with the study of the simpler case. It is this same consideration that made me restrict this study in the first instance to the stars within 30° of the galactic circle. By the exclusion of the higher latitudes we gain in homogeneity of material without losing many stars. It might even have been desirable to restrict our plan still more, for instance, by excluding the latitudes −20° to −30°, the longitudes beyond 330°, and the proper motions below a certain limit. The stars which appear to be exceptional would thus have nearly disappeared. I have preferred, however, not to go as far as this, but to cover a good part of the sky completely, in order to see what precision may now be obtained and also what difficulties remain to be faced.

The following short summary of the paper may make it more readily understood. It will at the same time supplement the information given in the paper itself.

All the helium stars between galactic latitudes ±30°, longitudes 216°–360°, in Boss's *Preliminary Catalogue*, which is complete to the sixth magnitude on Boss's scale (5.80 Harvard), but contains, besides, quite a number of fainter stars, are given at the end of the paper in three lists. The first two contain separately for galactic latitudes 0° to +30° and 0° to −30° all the stars having a secular motion of 1ʺ7 or greater. All the smaller proper motions are contained in the third list. The stars of the first two lists are plotted in Map 1, those of the third in Map 2. I have given the maps for the entire 360° of longitude, in order to show clearly the apparent tendency in these stars to clustering, a phenomenon that has been remarked by several astronomers. The most extensive of these "clusters" shown by Map 1 is between the longitudes approximately 200° and 340°. It is this group which forms the main subject of the present paper.

The total number of stars in Boss known to be of spectral class B is 752. Of these 655 (87 per cent) are between galactic

latitudes ±30°. The total number of B stars included in the present paper is 319, i.e., 42.4 per cent of all the Boss helium stars, and 48.7 per cent of those between galactic latitudes ±30°.

The first point is to determine, both by the proper motions and by the radial velocities, whether the group has a motion as a whole, and if so, the limits between which this is the case. This point is treated in detail in Sections 2–7. We meet with some difficulty in fixing the limit of the group on the side of the smaller longitudes. In order to arrive at a satisfactory conclusion about this point we

TABLE I

Group	l	b	100μ	n
A.	270° to 360°	0° to +30°	≦2″.4	96
B.	289 337	0 −30	3.0	29
C.	240 270	0 +25	1.7	28
D.	258 288	0 −30	1.7	20
E.	239 256	0 −20	1.7	25
F.	226 238	−20 +10	1.7	20
G.	217 225	−15 +10	≦1.7	11

TABLE II

Group	l	b	100μ	n
a.	195° to 216°	0° to −10°	≦1″.7	13
b.	195 216	−11 −25	1.7	6
c.	165 216	0 +30	1.7	9
d.	155 180	0 −14	1.7	11
e.	155 180	−15 −30	≦1.7	9

are compelled to include, as additional stars, those down to longitude 155°. The limit being fixed at 216°, these stars in lower longitudes are again discarded. They are of course contained in the maps, but not in the lists. The bulk of the stars retained were divided into the 7 groups indicated in Map 2, Plate II (at the end of this paper), and defined as in Table I (l=longitude; b=latitude; μ=total proper motion; n=number of stars).

The additional stars were subdivided as shown in Table II.

Some parts of the sky are not covered, but, as they contain very few stars, they may provisionally be disregarded. Only the

stars within the limits of Table I have been used in the derivation of the stream-elements. They have been marked in Lists 1 and 2 by their group-letters. The remaining stars are considered only when definitive elements have been obtained. The proper motions exceeding $1''.6$ per century in the eight regions *E, F, G, a, b, c, d, e* have been plotted in Fig. 1. If in each of these areas we compute the average direction of the proper motion we get what has been

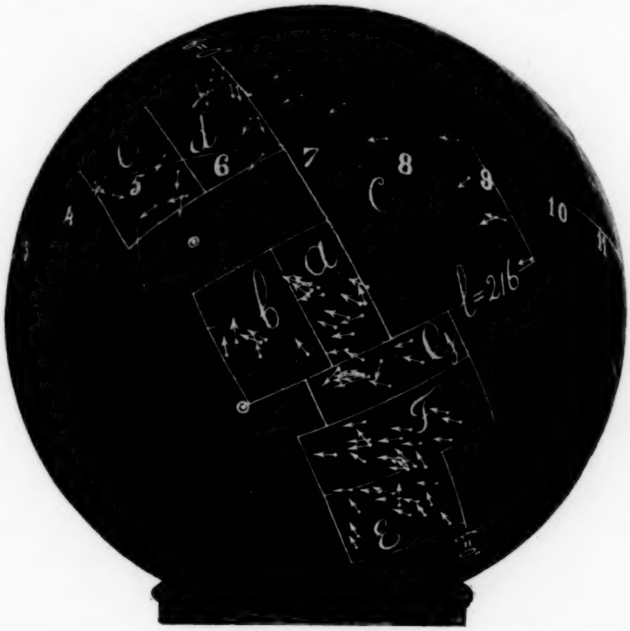


FIG. 1

represented in Fig. 2. Both figures make it plain that the directions of the proper motions converge, not toward a single point, but toward two rather widely separated points. A study of the figures further shows that the limit between the two streams thus indicated must lie somewhere near longitude 216°, and that there can be but little overlapping. Here, then, it was resolved to place the lower limit of the stars to be considered in the present study.

The community of motion in each of the parts *A*, *B*, ... *G* considered separately is very evident in our lists. It becomes

more striking if the proper motions are plotted. But is this motion the same for all the regions? In order to arrive at an unbiased judgment, an attempt is made to derive as complete elements as possible of the common motion in each part separately. The regions *E*, *F*, *G*, which are relatively small, are combined. In accordance with other investigations we have assumed from the outset that the observed radial velocities require a constant correction of −4.3 km. Campbell[1] finds −4.1. Fortunately our

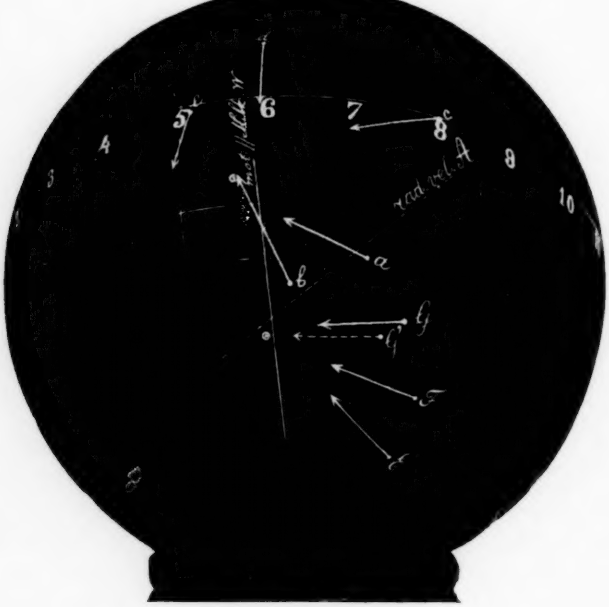


FIG. 2

adopted value is exactly confirmed by the final solution in Section 8. In the cases shown in Table III a more or less reliable result is obtained. The conclusion is that for these regions there is community of motion.

For the regions *C* and *D* the data are not sufficient for the derivation of perfectly independent elements. They lie, however, between the regions *A* and *B* on the one side, and the regions *E*+*F*+*G* on

[1] *Lick Bull.*, No. 196, p. 127, 1911.

the other (see Map 2); and both the position angles of the proper motions and the radial velocities agree with what is found from the elements furnished by surrounding regions. The conclusion is drawn, that, for the whole of the regions covered, the bulk of the stars move in a single stream whose elements are:

Vertex	$18^h18^m, +42°$	See (27)
Stream-velocity	$V = -18.3 \pm 0.9$ km	
Const. corr.	$K = -\ 4.3 \pm 0.5$ km	

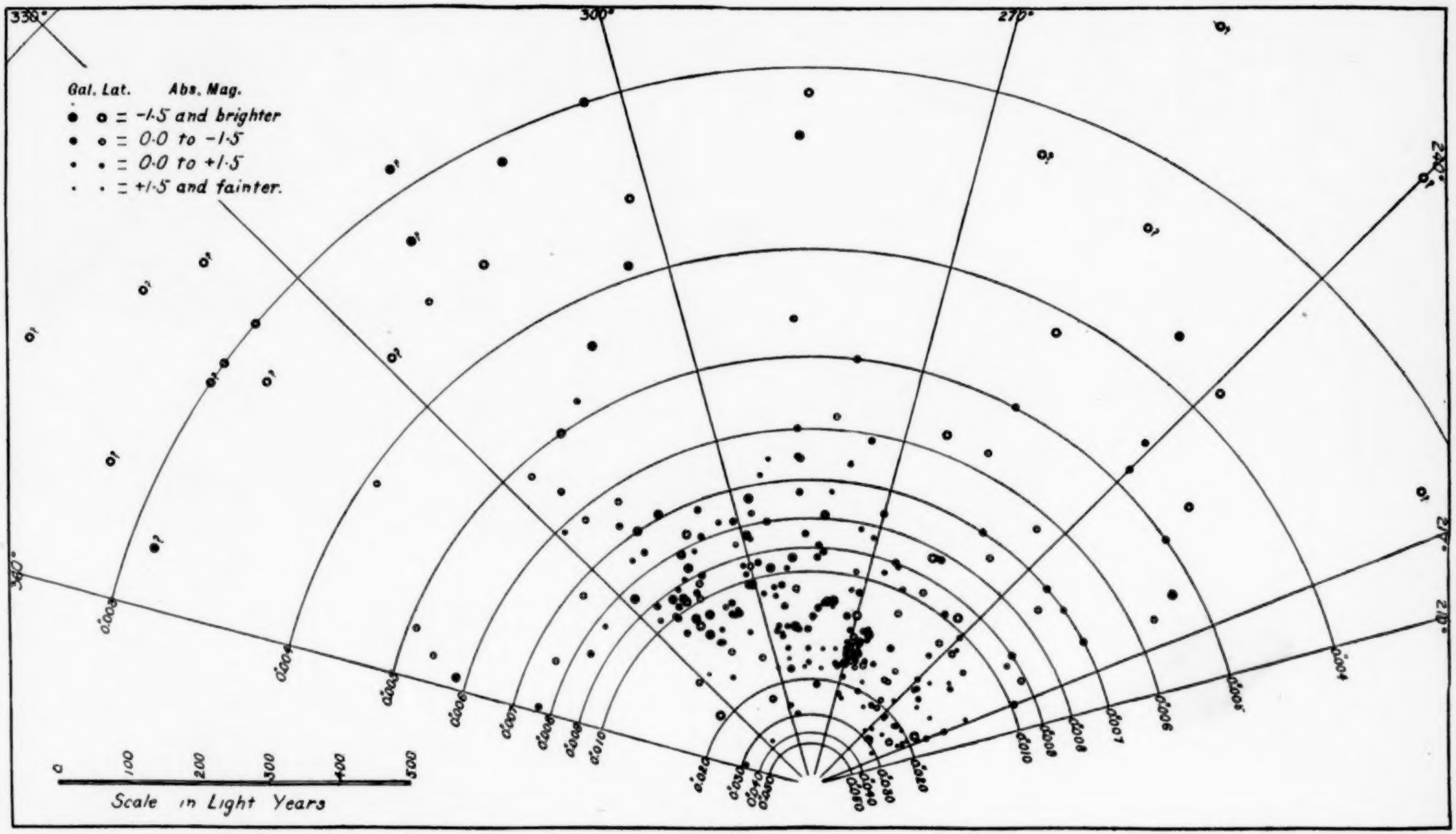
The residuals left by these elements in the mean position angles of the seven separate regions *A*, *B*, ... *G* are shown in Table XVIII, those in the radial velocities in Table XIX.

TABLE III

Region	Vertex		Stream-Velocity	P.E.	Reference Equations
A.	18^h23^m	$+41°.5$	-15.3 km	± 2.0	(1) and (2)
B.	18 30	$+37$	. . . .	. . . .	(5)
E+*F*+*G*.	17 58	$+41.5$	-19.1	± 0.75	(7) and (22)

The comparison of the motions of the individual stars, with the pure stream-motion according to these elements, leads to an estimate of the probable amount of the peculiar motion (Section 9). The probable amount r_u for the component in any direction is found to be only ± 2.1 km per second. It is the smallness of this amount which suggests the possibility of applying to our case the method used for finding the parallax of the members of such groups as those of the Hyades and Ursa Major. The necessary formulae both for the parallaxes and for their probable errors are given in Section 12. The results obtained for the helium stars are summarized in Section 13. They enable us to draw a map in space of these stars (Map 3). Particulars may be found in Section 14.

A few stars have been excluded from this computation of the parallax on the ground that they do not belong to the group. These "exceptional" stars have been considered in Sections 10 and 11. The conclusion is reached that within the limits adopted for the present paper—and not counting the Vela group—not more than 4 per cent of the whole number of stars are extraneous to the group,



MAP 3.—ARRANGEMENT OF HELIUM STARS IN SPACE

and these are mostly near the upper limit between 296° and 360° of galactic longitude and $-9°$ and $-30°$ of latitude. The Vela group, treated in Section 6, probably must be considered as a local group, much like the restricted Perseus group found by Eddington.[1] The data are as yet insufficient for the derivation of the elements of its motion.

The parallaxes for the stars of the main group once found, there is of course no theoretical difficulty in finding the luminosity-curve and the star-density at different distances from the sun. In practice it was found (Sections 15 and 16) that a somewhat satisfactory result could be obtained for the B0–B5 stars. That for the B5–B9 stars is less reliable. Still as the constants of the luminosity-curve for the B5–B9 stars were required for a particular purpose they have been deduced as well as the observations would permit. They are given in (48).

The main object of the paper, as far as the helium stars are concerned, would thus have been attained were it not that the proper motions which have been taken from Boss's ***Preliminary Catalogue*** were tacitly assumed to be systematically correct. For stars having in general such small proper motions as the B stars, the supposition is dangerous. A separate consideration of this point is therefore undertaken in Section 17, which leads to the result that a correction to Boss's proper motions in declination amounting to $+0''.003$ is clearly indicated. The change in the stream-elements produced by such a correction has been determined. The altered parallaxes are given in the last column of the lists.

Section 18 is concerned with the question in how far the same method for finding the individual parallaxes—or a modification of it—may be applied with success to stars of the other spectral classes. In this section, as in all that precede, the parallax is derived from the motion of the stars. This practically restricts the method—at least for the present—to the brighter stars. In the last two sections, therefore, is considered the question as to what data we may hope to obtain for the distances of the faint stars, for which our knowledge of the motions, both astronomical and spectroscopic, is too slender to be of much value.

[1] *Monthly Notices*, **71**, 43, 1910.

2. REGION A: LONG. 270° TO 360°; LAT. 0° TO +30°; 100 $\mu \geqq 2''.4$

Very small proper motions are rare in this region, particularly below longitude 300°, as is shown by Table IV. In fact, there are only 13 stars in all for which 100 $\mu < 2''.4$. We therefore begin by confining our attention to the 96 stars for which 100 $\mu \geqq 2''.4$.

TABLE IV

100 μ	l 270°–299°	l 300°–330°	n
0''.0 to 0''.9	....	3	3
1.0 1.9	2	6	8
2.0 2.9	8	9	17
3.0 3.9	13	17	30
4.0 4.9	14	11	25
5.0 5.9	16	1	17
6.0 6.9	6	2	8
8.2	1	....	1
12.1	1	....	1
Total	61	49	110

TABLE V

	l	b	100 μ	α	δ	100 μ	p	n
A I	270° to 306°	0° to +30°	2''.5 to 3''.9	14h23m	−45°.7	3''.30	219°.2	20
A II	270 306	0 +30	≧4.0	14 8	−46.7	5.50	223.6	39
A III	306 2	0 +30	≧2.4	16 23	−24.5	3.64	203.2	37
Mean (weight = number of stars)				15 4	−37.8	4.32	214.7	96

The parallelism of the astronomical proper motions of these stars is so great, and the radial velocities vary relatively so little (and on the whole regularly with the position in the sky), that there can be no serious question as to the community of motion for the group. Still, in order to dissipate whatever doubt might still exist, I have subdivided the group in three parts, for which the averages given in Table V were formed.[1]

Further, for the radial velocities see Table VI.

For the radial velocities the means were found in two different ways: first, by adopting as weight the number of stars observed

[1] In order to see how far the differences between the separate values are due to differences in position, compare with the values of p those computed by the help of the definitive elements in Tables XVII and XIX. Four stars, Boss 3458, 3699, 3716, 3929, were accidentally overlooked in forming the averages.

(the radial velocities marked (:) or depending on one observation receiving half-weight); second, by modifying the weights in such a way that the mean radial velocity became zero. From the second mean it follows at once that the vertex must lie exactly

$$90^\circ \text{ from the point } 15^h11^m, -35°.9 \qquad \text{(A)}$$

We may, however, obtain a result with somewhat greater weight by using the first mean value. Only in this case we have to assume a value for the stream-velocity. If we adopt -19 km per second, then from the last line but one in Table VI we conclude that, λ being

TABLE VI

	α	δ	$\rho - 4.3$	Weight	Modified Weight
A I	14^h10^m	$-45°.5$	$+4°.9$	8.5	6.0
A II	13 55	-46.0	$+3.55$	9	6.4
A III	16 46	-22.4	-5.8	9	9
Mean (actual weights)	14 57	-37.8	$+0.8$	26.5	....
Mean (modified weights)	15 11	-35.9	0.0	....	21.4

the distance to the vertex, $-19 \cos \lambda = +0.8$, whence $\lambda = 92°.4$. The uncertainty of the adopted stream-velocity is of hardly any importance, for we may vary it between 13.5 and 33 km without changing λ more than a degree. It seems preferable, therefore, to use the latter result on account of its greater weight. It defines a small circle

$$92°.4 \text{ from the point } 14^h57^m, -37°.8 \qquad \text{(B)}$$

on which the vertex must be situated.

Finally, therefore, if we treat the region as a whole, the vertex is determined by the great circle defined by the last line of Table V and the small circle (B). The intersection of the two, which of course is nearly at an angle of 90°, gives

$$18^h23^m, \quad +41°.5 \text{ (vertex A)} \qquad (1)$$

The determination by (A) and Table V, which is absolutely free from any supposition as to the stream-velocity, gives practically the same result.

I have also computed the vertex for the three subdivisions separately on the two assumptions $V=-19.0$ and $V=-25.0$ km. We find:

TABLE VII

VERTICES FOR SUBDIVISIONS

	$V=-19.0$		$V=-25.0$	
A I	18^h18^m	$+44°$	$18^h\ 6^m$	$+41°$
A II	18 6	+38	17 54	+36
A III	18 33	+45	18 48	+49

The influence of a wrong assumption for the stream-velocity is thus seen to be relatively small in all cases. Considering the uncertainty of the radial velocities, the differences in the positions of the three vertices, especially for $V=-19.0$, is also gratifyingly small. There thus seems hardly a doubt possible that the three form part of a single cosmical group, moving in the direction defined by the vertex (1).

This point being settled, we can derive the stream-velocity. As the region is so limited this determination will of course be rather weak. Using all the radial velocities and applying the constant correction -4.3 km, I find:

$$V=-15.3\pm 2.0 \text{ km} \tag{2}$$

The residuals furnish for the probable deviation of a radial velocity resting on more than one observation the value:

$$r_\rho=\pm 3.4 \text{ km} \tag{3}$$

If we assume the position of the vertex to be correct, V will be but little dependent on the assumed constant correction. Neglecting the correction altogether, we should find $V=-16.9$. But the position of the vertex itself is dependent on the correction. Supposing it to be -5.3 km instead of -4.3, the position becomes 18^h15^m, $+39°$ with $V=-17.1$. It thus appears that any admissible change in K does not affect very seriously either the position of the vertex or the velocity V.

3. REGION B: LONG. 289° TO 337°; LAT. 0° TO −30°; $100\,\mu \geqq 3\overset{''}{.}0$

The parallelism of the motions is not so marked as in the preceding region as the summary in Table VIII shows at a glance. The two largest values included in the last column, though belonging to stars with considerable proper motions, Boss 4599 and 4913, have but little reliability because of the exceptionally great uncertainty in Boss's determination (compare with the probable errors given in Boss's catalogue). But even retaining these

TABLE VIII

p = position angle of proper motion	$100\,\mu$		
	$\leqq 1\overset{''}{.}6$	$1\overset{''}{.}7$–$2\overset{''}{.}9$	$\geqq 3\overset{''}{.}0$
<100°	2	1	1
100° to 120°	1	1	.....
120 140	.....	1	1
140 160	2	2	.....
160 180	.....	2	5
180 200	3	4	9
200 220	3	1	10
220 240	1	2	1
>240°	1	.....	2
Totals	13	14	29

we find that for the stars with secular proper motions exceeding 3″, 24 out of 29 values of p show deviations but little larger than can be accounted for by their probable errors r_p (see List 2).

For the smaller proper motions, where the accidental errors of the p's are of course more considerable, we find much the same state of affairs. There can be no doubt that, here again, we have to do with a very decided common motion. Still, in order to introduce as little uncertainty as possible, I will restrict myself provisionally to the proper motions exceeding 3″ per century. In taking the mean, it is nearly immaterial whether we retain or reject the five most divergent values. I decided to reject them and found:

α	δ	$100\,\mu$	p	P.E.	n	
17^h45^m	$-46\overset{\circ}{.}2$	$5\overset{''}{.}06$	$189\overset{\circ}{.}1$	$\pm 3\overset{\circ}{.}5$	24*	(4)

* In order not to produce exaggerated notions of the accuracy obtained I used the probable error found when we include the five rejected stars.

For the radial velocities the data are extremely poor. To enable the reader to judge the value of what we have, it seems necessary to write them out in full. See Table IX.

TABLE IX

Boss No.	α	δ	100μ	$\rho-4.3$	Remarks	Weight
4094........	$16^h\ 1^m$	$-58°$	6.″5	+ 1.7	1 obs.	0.5
4249........	16 38	−58	3.0	− 4.3	1 "	0.5
4156........	16 15	−49	3.3	− 3.3	1 "	0.5
4431........	17 24	−50	9.0	− 2.3	S.B. est.	0.5
4565........	17 59	−50	3.0	− 1.5	....	1
4657........	18 20	−46	5.3	− 5.1	....	1
4429........	17 24	−37	4.2	+12.7	S.B. est. rej.	....
4439........	17 27	−37	3.6	− 1.3	1 obs.	0.5
4737........	18 38	−36	5.3	− 2.9	....	1
4784........	18 49	−26	6.6	− 5.3	....	1
Mean...	17 47	−43.7	5.06	−3.01±1.35	....	6.5

The result for Boss 4429 (υ Scorpii) was rejected. According to Campbell the star is a spectroscopic binary. The estimated value of the velocity of the center of mass may be greatly in error or the star may belong to the second stream, in which the velocity fits well. The position angle of the proper motion, however, does not favor this view. The probable error given for the mean was derived from the value (3) found in the preceding section. The mean velocity of -3 km is sufficiently small to allow the application of the method followed in the determination of the vertex for the region A.

Assuming $V=-19.0$, we find that for the mean position of Table IX $\lambda=80.°9 \pm 4.°1$. This value defines a small circle on which the vertex must lie; (4) likewise defines a great circle. The vertex must be at the intersection of the two. We thus get the position:

$$18^h 30^m, \quad +37° \text{ (vertex } B\text{)} \tag{5}$$

which lies at a distance of not quite 5° from the vertex A (1). The divergence is fully accounted for by the probable errors.

Conclusion.—The stars of region B have a common motion which coincides with that of region A. The two regions together give a determination of the vertex, independent of radial velocities. It

is determined by the intersection of the directions defined by Table V and (4). I find

$$18^h33^m, \quad +45^\circ \text{ (vertex } A+B\text{, astronomical motion)} \qquad (6)$$

Owing to the smallness of the angle of intersection, which is about 30°, the position is not a strong one and may easily be in error by 6° or 7°. The agreement with (1) and (5), especially with the former, which is much the better of the two, is therefore all that can be desired. It may be taken as further proof—if such were required—of the identity of the stream-motions in the two regions.

4. REGIONS LONG. 217° TO 250°; LAT. −20° TO +10°; $100\mu \geqq 1''.7$ AND LONG. 155° TO 216°; LAT. −30° TO +30°; $100\mu \geqq 1''.7$

There is a sudden jump in the values of the *p*'s, at longitude 216°. The phenomenon seems so remarkable and for the present investigation so important that I have prepared Fig. 1 (p. 47), which shows for the region the proper motions of more than 1″.6 per century. Together with Fig. 2 (p. 48), showing the average motions, it will make the matter much more evident than any verbal description.

There is plenty of evidence that we are near some vertex. Because of the rapid change in position angle near such points, which has a tendency to obscure the phenomenon in question, it becomes necessary to subdivide. The choice has to be made with some care. The immediate neighborhood of any point of convergence in which all systematic motion vanishes has to be avoided. The following subdivisions were finally adopted. They leave small parts of the sky uncovered. As they contain but few stars they may provisionally be left out of consideration. (See also Map 2.)

TABLE X

Region	*l*		*b*	
E	256° to	239°	−20° to	0°
F	238	226	−20	+10
G	225	217	−15	+10
a	216	195	0	−10
b	216	195	−11	−25
c	216	165	0	+30
d	180	155	0	−14
e	180	155	−15	−30

Taking into account only the stars of which the centennial motion exceeds 1″.6 we obtain the means in Table XI.

From the normal place for *F* the six stars, Boss 2095, 2166, 2171, 2191, 2360, 2382, were excluded. Together with a few other stars they seem to belong to a separate group which will presently be considered under the name of the Vela group (see Section 6). From each of the regions *G*, *a*, *d*, one star—Boss 2060, 1958, 1567—was excluded because of the exceptional direction of its motion. From region *b* Boss 1401 was omitted. Apparently it belongs in region *G*. Finally from *c* Boss 1935 was excluded. Its radial velocity would

TABLE XI

Region	Proper Motions					Radial Velocities			
	α	δ	100μ	p	n	α	δ	$\rho-4.3$	Weight
E.......	8^h38^m	$-60°.2$	3″.3	292°.5	25	8^h45^m	$-59°.6$	+16.5	10
F.......	8 27	−48.8	3.2	270.3	20	8 38	−48.5	+16.6	5
G.......	7 51	−37.1	3.0	254.2	11	7 36	−38.0	+19.8	1
a.......	7 16	−27.9	3.2	286.5	13	7 35	−27.0	+18.7	1
b.......	6 18	−33.2	4.2	333.5	6	6 1	−34.0	+20.7	1.5
c.......	8 0	− 0.3	3.4	249.9	9	7 36	+ 6.0	+14.5	3
d.......	5 54	+13.2	3.0	178.5	11	5 50	+14.3	+17.8	3
e.......	5 7	− 1.6	2.3	196.5	9	5 2	0.0	+15.0	3

assign it rather a place in the second stream. This star will be further discussed in a subsequent paper to be devoted to the second stream of helium stars. Boss 1994 in *a* ought probably to be placed in *G*. As, however, little would be altered by the change, I have not thought it worth while to make it.

The directions defined by the averages in Table XI have been plotted in Fig. 2 (p. 48). It shows at once that the proper motions of the regions *E*, *F*, *G* converge very nearly to a single point. The same is the case for the proper motions in *a*, *b*, *c*, *d*. The points of convergence, however, are not the same in the two cases, but lie pretty far apart. I find:

Regions	Convergent	Distance	
E, F, G	$5^h58^m, -41°.5$	1°, 1°, 2°	(7)
a, b, c, d	5 40 , −15	3.5, 0, 3, 3	(8)

In both cases the convergence toward a single point is extremely close, as appears from the above distance at which the several

directions pass these points. The direction for *e*, however, passes the point (8) at a distance of 12°. Five of the nine stars in the region have motions passing close to the convergent. If, excluding no star, we also take the direction for *e* into account, the convergent (8) is changed to:

Regions	Convergent	Distance	
a, *b*, *c*, *d*, *e*	5^h27^m, $-12°.5$	4°, 2°.5, 0°, 7°, 8°	(9)

which lies still farther from the point (7).

Altogether I think that the evidence for the close convergence of the four directions *a*, *b*, *c*, *d*, or the five directions *a*, *b*, *c*, *d*, *e*, to a single point not far from (8) is absolutely convincing. It is scarcely less evident that the directions for *E*, *F*, *G* do *not* pass through (8). Their distances from this point are: *E*, 11° (25 stars); *F*, 21°.5 (20 stars); *G*, 25° (11 stars).

Most notable, perhaps, is the divergence for *F*, containing 20 stars. As has been said, six stars of the Vela group were here excluded. Had these been included, the distance would have been *increased* to 32°.5. Moreover, the convergence of the motions in *E*, *F*, *G* to the point (7) is almost perfect. Finally, this point is practically identical with the convergent of the regions *A* and *B*, which, as we shall see presently, has also to be adopted for the regions *C* and *D*. As the regions *E*, *F*, *G* form one continuous group in the sky with *A*, *B*, *C*, *D* (see Map 2), this coincidence of the convergents presents nothing surprising. All this, I think, leads inevitably to the conclusion that the stars of the regions on both sides of galactic longitude 217°, approximately, form two distinct groups.

I am fully aware that Boss's catalogue, from which the proper motions were taken, may be subject to systematic errors, and that these may have altered the direction of the proper motions in such a way that by correcting them the convergents (7) and (8) or (7) and (9) would be brought nearer together. May they not be brought into exact coincidence? I shall consider the question of systematic errors in Section 17, and the conclusion will be reached that there is really an indication of systematic error in the proper motions in declination. If the regions *a*, *b*, *c*, *d*, *e* do not

require the same correction, the removal of the error will indeed bring the convergent (8) nearer to (7) but by about $3°$ only.

The stars of the regions *a, b, c, d, e* have not yet been investigated for systematic error. Meanwhile, it is evident that to explain a difference of 80°, in the position angles of the proper motions in such close-lying regions as *G* and $a+b$ (see Fig. 1, p. 47) by difference of systematic error for the two regions is wholly out of the question. It may be urged that at best the normal for region *G* rests on too small a number of stars (11) to be wholly convincing. In order to see what value this objection has, I have drawn on the early A stars of this region. If, in order to make the material more comparable with the B stars, we exclude the two objects whose secular proper motions exceed 10″, we find eleven stars between the limits $\alpha = 7^h0^m$ and 8^h10^m and $\delta = -35°$ and $-48°$. The region nearly coincides with *G*, and gives the average

$$7^h35^m,\ -40°.1;\quad 100\,\mu = 3''.0;\quad p = 260°.0 \text{ (11 A0–A3 stars)} \qquad (10)$$

The great circle defined by this direction has been drawn as a dotted line in Fig. 2 (p. 48). It confirms absolutely what was found from the B stars. If we exclude the star Boss 1826, which is almost at the antivertex itself, the direction would tend to throw this point even farther toward the south. A similar computation for region *F* gave

$$8^h48^m,\ -47°.6;\quad 100\,\mu = 3''.6;\quad p = 267°.7 \text{ (13 A0–A3 stars)} \qquad (11)$$

which also agrees well with that in Table XI.

There is a consideration of quite another kind which still further confirms the practical correctness of the position (7) found for the antivertex of the stars *E, F, G*. At the January 1912 meeting of the Academy of Sciences in Amsterdam, I presented a paper in which it was shown that, as far as can be judged from existing data, the motions of all the somewhat richer star groups relative to the center of gravity of the whole stellar system are at least approximately parallel to the plane of the Milky Way. From this communication the following lines are taken:

Let

h = yearly motion of the solar system relative to the center of gravity of the stellar system.

β = galactic latitude of the apex of this motion.
v = yearly linear motion of a star group relative to the same center as h.
b = galactic latitude of its *true* antivertex.
V = yearly linear motion of this same group with respect to the sun.
B = galactic latitude of its *apparent* antivertex.

The motion v is the resultant of V and $-h$. Therefore, projecting on a normal to the Milky Way

$$V \sin B = v \sin b - h \sin \beta \tag{12}$$

We assume for the apex[1] $\alpha = 269^\circ.0$, $\delta = +32^\circ.0$, whence $\beta = +23^\circ$, and take $h = 19.5$ km per second (Campbell). Then, if the motion of the group is parallel to the Milky Way, we find from (12) as the condition for this parallelism

$$V \sin B = -7.6 \tag{13}$$

For the best values of the elements known to me we have:

TABLE XII

	V	$V \sin B$
Hyades	45.6 km	− 3.4 km
Ursa Major	18.4	−11.1
Scorp.-Centaur. (=A)	18.8	− 6.7
Perseus	18.0	− 4.1
2d type, 1st stream	32.6	− 8.4
A stars, 1st stream	27.7	− 6.1
B stars, 1st stream	22.0	− 6.7
2d type, 2d stream	18.4	− 8.5
A stars, 2d stream	24.5	−10.2
Mean	....	− 7.25

Considering the uncertainties, not only in the elements of the several groups, but also in the position of the Milky Way, the agreement seems very close. If now we assume that equation (13) also holds for the group $E+F+G$, we obtain a new datum for the determination of the stream-elements. Combining the mean values in Table XI found for ρ in the three separate groups E, F, G, we have

α	δ	b'	$\rho - 4.3$	
$8^h 38^m.5$	$-54^\circ.8$	$-8^\circ.7$	$+16.74$ km (weight 16)	(14)

[1] *Groningen Publication*, No. 21, 1908.

Let λ be the distance of this point from the antivertex, then

$$V \cos \lambda = 16.74 \tag{15}$$

This equation combined with (13) gives

$$\frac{\cos \lambda}{\sin B} = -2.20 \tag{16}$$

It is easily proved that the locus of the points on the sphere which satisfy this condition is a great circle at right angles to the arc connecting the point (14) with the pole of the Galaxy and intersecting this arc at a distance d from (14) determined by

$$\tan d = -\frac{7.6 + 16.74 \sin b'}{16.74 \cos b'} \tag{17}$$

from which

$$d = -18.^\circ 5 \tag{18}$$

The great circle passes through the points (read from a globe) $6^h 18^m$, $-60°$; $6^h 3^m$, $-40°$; $5^h 57^m$, $-20°$. It is shown in Fig. 2 and we see that it cuts the average directions of the proper motion in E, F, G (Table XI) under favorable angles. The four circles intersect almost in a point. In fact, they all pass through a circle with a radius of barely 1° round the point $6^h 0^m$, $-41°$. The convergent (7) is thus strongly confirmed. The figure, finally, shows also the small circle (B) of Section 2, on which, according to the radial velocities, the antivertex for the region A must lie. It passes almost through the convergent (7) whereas the distance from (8) is about 22°.

5. STREAM-VELOCITY OF E+F+G

Assuming the correctness of the constant correction K, for which we used -4.3 km, the determination of the stream-velocity—owing to the nearness of the antivertex—must be relatively good, at least if we take into account the very limited number of stars with given radial velocity.

A small change in the position of the vertex will hardly affect the velocity. I will adopt the position

$$18^h 20^m, \quad +42° \tag{19}$$

which is that best satisfying the directions defined by Table V (4), and Table XI. In fact I find that these directions pass the point (19) at the following distances:

$$\left.\begin{array}{lll} A & \text{at } 0^\circ\!.5 & (96 \text{ stars}) \\ B & 2.0 & (24 \text{ “ }) \\ E & 2.0 & (25 \text{ “ }) \\ F & 2.4 & (20 \text{ “ }) \\ G & 2.5 & (11 \text{ “ }) \end{array}\right\} \quad (20)$$

The angles of intersection, taken from the globe are roughly

$$\left.\begin{array}{lll} & F & G \\ A & 29^\circ & 53^\circ \\ B & 60 & 85 \end{array}\right\} \quad (21)$$

The determination (19) is quite independent of the radial velocities and must be fairly good. With it the radial velocities of Table XI furnish in the usual way

$$V = -19.1 \pm 0.75 \text{ km} \quad (22)$$

The probable error of a single determination of radial velocity, resting on more than a single measurement and still including peculiar motions, is found to be

$$r_p' = \pm 2.6 \quad (16 \text{ full-weight stars}) \quad (23)$$

The velocity (22) and the convergent (7) agree so closely with the corresponding values (2), (1), and (5) that we may conclude, with a certainty as great as the observations will allow, that the *stars in the regions E, F, G partake in the motion of A and B.*

6. THE VELA GROUP

We have already had occasion to refer to a group of stars in the constellation Vela, which stands out pretty clearly from the other stars. Rather less than half of the stars in the region

$$\left.\begin{array}{ll} l & b \\ 230^\circ \text{ to } 235^\circ & -10^\circ \text{ to } \;\; 0^\circ \\ 236 \quad\;\; 265 & -\;5 \quad\;\; +5 \end{array}\right\} \quad (P)$$

seem to belong to it.

In order to show what indication there is of such a group I computed the values of the divergences $p-p_0$ of the angle of position p from the angle of position p_0 of the great circle from star to antivertex, taking for the vertex a point near (19). This was done not only for the stars in region P, but also, as a comparison, for the adjacent region Q in higher and lower latitudes (remaining part of the region long. 230° to 265°; lat. −20° to +20°).

TABLE XIII

NUMBER OF STARS

$p-p_0$	P	Q	$p-p_0$	P	Q
−90° to −99°	1	....	−10° to −19°	6	4
−80 −89	....	....	0 − 9	3	3
−70 −79	4	....	+ 1 + 9	2	8
−60 −69	3	....	+10 +19	2	6
−50 −59	2	1	+20 +29	3	1
−40 −49	2	1	+30 +39	2	....
−30 −39	2	3	+40 +49	....	1
−20 −29	3	2			
			Total	35	30

TABLE XIV

Boss. No.	Sp.	Mag.	l	b	$100\,\mu$	p	r_p	p_c	O−C
2096....	B8	5.4	219°	− 3°	3″.7	189°	12°	244°	−55°
2095....	B1	4.3	229	−10	1.8	186	17	283	−97
2166....	B3	4.8	231	− 7	1.7	205	17	276	−71
2171....	B3	5.4	231	− 7	2.6	198	22	278	−80
2191....	B3	5.3	231	− 6	3.8	208	21	275	−67
2360....	B5c	5.5	233	− 1	3.4	195	13	264	−69
2382....	B0	4.9	234	− 1	2.3	211	22	264	−53
2577....	B8	5.2	243	+ 1	3.1	231	17	266	−35
2659....	B0	6.0	245	+ 3	1.7	216	28	263	−47
2669....	B0	6.5	245	+ 3	3.2	196	15	262	−66
2843....	B8	6.6	253	+ 3	1.7	211	25	259	−48
2860....	B3c	5.4	255	0	2.0	186	22	264	−78
2876....	B5	5.5	257	− 4	3.8	209	13	269	−60
3127....	B5	5.7	264	0	1.9	201	26	251	−50
3399....	Oap	5.6	272	− 2	2.6	203	15	241	−38

We see that whereas the region Q shows a single maximum at about $p-p_0=0$, P shows a double maximum; further, in P there are 26 negative and 9 positive values, whereas in Q the numbers are nearly equal. The stars given in Table XIV ought probably to be considered as members of the group.

The values of p_c (computed position angle) were computed with the definitive vertex found in Section 8. The extent in galactic longitude is relatively great; in latitude it is small. In this respect the group is much like the restricted Perseus group found by Eddington.[1] On the whole the reality of the group seems to me probable, though not beyond a doubt. The range in proper motion seems somewhat in excess of what might be expected in a physical group. Meanwhile, the probable errors are all rather large, so that the differences may very well be spurious in great part. In any case it would be very unsafe to include these stars in a discussion of the general group. The data are not sufficient for a reliable determination of the vertex and the group-velocity.

It might be different if good determinations of radial velocity were known for all or for a great part of the stars.[2] Such determinations might very possibly establish the reality of the group beyond a doubt.

7. REGION C: LONG. 240° TO 270°; LAT. 0° TO +25°; 100 $\mu \geqq 1''7$ REGION D: LONG. 258° TO 288°; LAT. −30° TO 0°; 100 $\mu \geqq 1''7$

Leaving out the six Vela stars within these limits, we get the averages given in Tables XV and XVI.

The quite abnormal radial velocity for Boss 3115 (List 1), depending on a single observation, has been rejected. The probable

TABLE XV

Region	α	δ	100μ	p	P.E.	n	p Comp. by (19)	O−C
C.	11^h52^m	−50°4	4″36	248°2	±2°2	28	243°9	+4°3
D.	12 43	−68.6	3.46	243.0	3.2	20	246.8	−3.8

TABLE XVI

Region	α	δ	100μ	$\rho-4.3$	P.E.	n	ρ Comp. by (22)	O−C
C.	11^h45^m	−55°1	. . . .	+10.5	±1.3	8	+11.65	−1.15
D.	11 47	−68.4	. . . .	+13.9	1.6	4.5	+12.53	+1.37

[1] *Monthly Notices*, **71**, 43, 1910.

[2] As far as I know there is only one observation available, a single measure of Boss 2860.

errors of the p's have been found by comparing in each region the individual values with the means for that region; those of the ρ's by assuming in accordance with (3) the probable error of one observation to be ± 3.4 km.

From the above data it seems impossible to derive reliable values of both the vertex and the stream-velocity. As, however, C and D lie between the regions A and B on the one side, and E, F, G on the other, and as further all the regions A, B, ... G form one continuous group in the sky, there is a strong presumption in favor of the view that both C and D belong to the same system as A, B, E, F, G. This presumption will give way to conviction if we find that both the position angle of the proper motions and the amount of the radial velocities agree with the values furnished by the stream-elements (19) and (22) for A, B, E, F, G. This proves really to be the case within the limits assigned by the probable errors; see Tables XV and XVI. For the general mean of the two regions together the agreement with these stream-elements is almost perfect.

We are thus driven to the belief that the stars in C and D join in the common motion of the regions A, B, E, F, G, so that finally we reach the conclusion that *the stars in the whole of the regions A, B, ... G have a common motion.* Provisionally this conclusion has of course to be restricted to the stars which have thus far contributed to our result, and even of these a few may have to be excluded.

8. DEFINITIVE ELEMENTS OF THE WHOLE GROUP A+B+C+D+E+F+G

Knowing this, we can now apply all of our data to the derivation of definitive elements for the greater group. If the constant correction K of the radial velocities were known with all desirable precision, it might be best to combine all the data furnished by proper motions and radial velocities for the determination of the direction and the amount of the stream-motion. Such, however, is not the case, and it is necessary to include the constant K among the unknowns of the problem. This being so, and the astronomical proper motions being by themselves sufficient for a satisfactory determination of the direction of motion, it seems

preferable not to make use of the radial velocities at all in that determination. I therefore simply drew on a globe the great circles defined by the data already given, but which, for clearness, are here repeated in Tables XVII and XVIII.

TABLE XVII

Region	α	δ	100μ	p_0	P.E.	n	p_c	O−C	$\frac{(O-C)}{\sin\lambda}$
A I......	14^h23^m	$-45°.7$	$3''.3$	$219°.2$	....	20	$220°.5$	$-1°.3$	$1°.3$
A II.....	14 8	−46.7	5.5	223.6	....	39	222.9	+0.7	0.7
A III....	16 23	−24.5	3.6	203.2	....	37	202.1	+1.1	1.1

TABLE XVIII

Region	α	δ	100μ	p_0	P.E.	n	p_c	O−C	$\frac{(O-C)}{\sin\lambda}$
A........	$15^h\ 4^m$	$-37°.8$	$4''.3$	$214°.7$	....	96	$213°.8$	$+0°.9$	$0°.9$
B........	17 45	−46.2	5.1	189.1	±3.5	24	186.2	+2.9	2.9
C........	11 52	−50.4	4.4	248.2	±2.2	28	243.9	+4.3	3.5
D........	12 43	−68.6	3.5	243.0	±3.2	20	246.6	−3.6	2.9
E........	8 38	−60.2	3.3	292.5	....	25	294.8	−2.3	1.1
F........	8 27	−48.8	3.2	270.3	....	20	274.6	−4.3	1.7
G........	7 51	−37.1	3.0	254.2	....	11	247.5	+6.7	2.2
G'.......	7.35	−40.1	3.0	260.0	....	11	256.4	+3.6	1.2

Because of the small number of stars in the important normal *G* I have added the normal *G'* (10) furnished by the early A stars of this region. All the directions pass through a circle with a radius somewhat greater than 3°, having its center at

$$18^h18^m,\quad +42° \text{ (definitive vertex)} \qquad (24)$$

This point was adopted as the definitive vertex of the whole group. The several directions pass it at distances given in the last column. The last column but one shows the divergences of the observed angles of position from the position angles of the circles passing through the vertex. The numbers of the last column show that, considering accidental errors alone, it is very unlikely that the position (24) is in error by more than 2°. The probable error derived in the usual way would certainly be found smaller than this.

The position of the vertex being known, we can now find the stream-velocity V from the whole of our data. Including as an unknown the constant correction K which must be applied to the observed velocities, the equations of condition take the form

$$V \cos \lambda - K = \rho$$

Each star of which the radial velocity has been determined gives an equation of this form. There are 74 such stars in all. Of these I rejected, as before, Boss 4429 and 3115. Eighteen stars, a few of which have been marked as very uncertain by the observers, the rest depending on a single observation, have been given half-weight. Three of the latter class were given weight 1/3 in order that all the observations with diminished weight might be combined to full-weight values. I thus obtained 61 equations of condition of equal weight. Solved by least squares they give

$$\left.\begin{aligned} V &= -18.21 \pm 0.87 \text{ km} \\ K &= -\ \ 4.38 \pm 0.50 \end{aligned}\right\} \quad (25)$$

The probable error of a full-weight observed radial velocity was found to be:

$$r'_\rho = \pm 2.95 \text{ km} \qquad (26)$$

If the value of K were assumed to be known a priori, the probable error of V would be lowered to ± 0.68 km. This, however, seems to me unwarranted. It may even be said that the present determination, depending on the discussion of a collection of stars which form a single group, is the only quite unobjectionable one given up to the present. In conclusion I will adopt as the final elements of the whole group:

$$\left.\begin{aligned} &\text{Position of vertex (1900)} \quad 18^h 18^m, \quad +42^\circ \\ &V = -18.3 \pm 0.9, \quad K = -4.3 \pm 0.5 \end{aligned}\right\} \quad (27)$$

The representation of the single observations, both in position angle and in radial velocity, is shown in Lists 1, 2, 3. For the normals of position angle it has been shown in Table XVIII. For those in radial velocity it may be seen from the summary given in Table XIX.

The agreement between observation and computation leaves nothing to be desired. The last column, O−C′, will be explained in Section 17.

TABLE XIX

Region	α	δ	$\rho_0 - 4.3$	Weight	ρ_c	O−C	O−C′
A I......	14^h10^m	−45°.5	+ 4.9	8.5	+ 4.3	+0.6	+0.8
A II.....	13 55	−46.0	+ 3.55	9	+ 4.9	−1.35	−1.4
A III.....	16 46	−22.4	− 5.8	9	− 6.9	+1.1	+1.2
A........	14 57	−37.8	+ 0.8	26.5	+ 0.6	+0.2	+0.2
B........	17 47	−43.7	− 3.0	6.5	− 1.3	−1.7	−1.3
C........	11 45	−55.1	+10.5	8	+11.2	−0.7	−0.6
D........	11 47	−68.4	+13.9	4.5	+12.1	+1.8	+2.1
E........	8 45	−59.6	+16.5	10	+16.1	+0.4	+0.5
F........	8 38	−48.5	+16.6	5	+16.6	0.0	−0.2
G........	7 36	−38	+19.8	1	+17.6	+2.2	+1.9

9. PROBABLE ERRORS AND PROBABLE AMOUNT OF PECULIAR MOTION

The value (26) of r_ρ' evidently includes both the error of observation and the peculiar motion. It is desirable to find the probable amount of each of the two separately. In Fig. 3, which is supposed

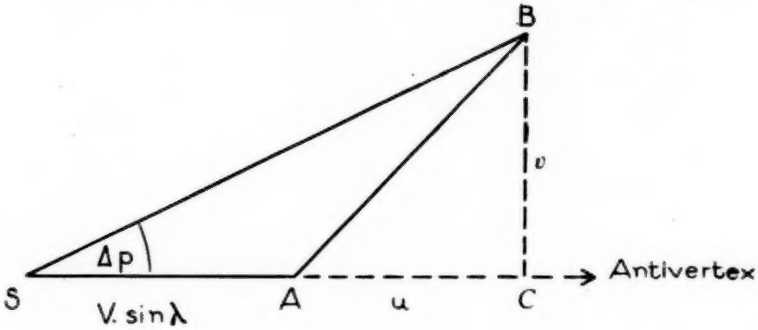


Fig. 3

to be in a plane at right angles to the line of sight, let S be any star; $V \sin \lambda$ the projection on the plane of the figure of the stream-velocity; u and v the components in this same plane of the star's linear peculiar motion. SB will be the total linear motion projected on this plane. It is this motion which is seen by the observer

under the angle μ, the total astronomical motion. Let finally Δp be the angle of *SB* with the great circle toward the antivertex.

It has been assumed—as a provisional hypothesis, to be replaced later by a real determination—that the peculiar motions are distributed according to Maxwell's law. Schwarzschild has already found that the assumption cannot be quite correct, but for the present purpose we cannot be led into serious error by adhering to it. I shall therefore suppose the law to hold. From this it follows that the quantities u and v will be distributed according to the error law. We may thus assume that

$$\text{Frequency of } u \text{ between } u \text{ and } u+du=\frac{h'}{\sqrt{\pi}}e^{-h'^2u^2}du \tag{28}$$

For v we have the same frequency law. From the figure we see that

$$(V\sin\lambda+u)\tan\Delta p=v \tag{29}$$

The stars near the vertex or antivertex cannot well serve for the present purpose. We shall therefore consider only stars for which $\sin\lambda$ is considerable For such stars, if they belong to the spectral class B, u will practically always be smaller than $V\sin\lambda$, so that, V being taken positive, the parenthesis in (29) will be positive. Consequently $\tan\Delta p$ and v will be of the same sign. If, therefore $\bar{v}$ and $\overline{\tan\Delta p}$ represent the averages of v and $\tan\Delta p$ respectively, all taken positively, and if we suppose that we have to do with a large number of stars all at the same distance λ from the vertex, we shall have

$$\bar{v}=V\sin\lambda\,\overline{\tan\Delta p}+\text{average value of }(u\tan\Delta p)$$

As the sign of u will be as often positive as negative and is independent of Δp, the last term will be vanishing. As further $\bar{u}$ is of course equal to $\bar{v}$ we find

$$\bar{u}=\bar{v}=V\sin\lambda\,\overline{\tan\Delta p} \tag{30}$$

This equation enables us to find the average values $\bar{u}$ and $\bar{v}$ from observed quantities, at least on the supposition that Δp is free from observation error. In order to satisfy this condition with at least some approximation, I took from the catalogue at the end of this paper the values of Δp of only those stars for which the

probable error r_p of the position angle, consequently of Δp, does not exceed $10°$. I thus found

	$\sin\lambda$	$\overline{\Delta p}$	
	0.90 to 1.00	$11^\circ\!.1$ (101 stars)	
	0.80 0.89	9.8 (21 stars)	(31)
	0.70 0.79	14.1 (9 stars)	
Means	$\sin\lambda=0.927$	$\overline{\Delta p}=11^\circ\!.1$ (131 stars)	

This value must now be cleared of the effect of observation error. The probable errors, r_p, of the position angles have been inserted in Lists 1, 2, 3, at the end of this paper.[1] I find as the average value for the stars used, $r_p=\pm 6^\circ\!.3$, to which corresponds an average deviation of $\pm 7^\circ\!.5$. Therefore, freed of observation error,

$$\overline{\Delta p}=\sqrt{(11.1)^2-(7.5)^2}=8^\circ\!.2 \tag{32}$$

Finally, by (30), if we assume $\overline{\tan\Delta p}=\tan\overline{\Delta p}$, which cannot be appreciably in error, $\bar{u}=\bar{v}=2.48$ km, to which corresponds the probable amount of u or v,

$$r_u=r_v=\pm 2.1 \text{ km per sec} \tag{33}$$

If now r_ρ represents the true probable error of a radial velocity observation resting on more than one observation, *excluding* peculiar motion, we shall have (r_u representing the probable amount of the peculiar motion in any direction, therefore also in the line of sight)

$$r_\rho=\sqrt{r_\rho'^2-r_u^2} \tag{34}$$

Therefore by (26) and (33)

$$r_\rho=\pm 2.1 \text{ km per sec} \tag{35}$$

10. STARS BETWEEN GALACTIC LONG. 217° AND 360°, LAT. ±30°, $100\,\mu \geqq 1''\!.7$, WHICH DIVERGE MARKEDLY FROM THE STREAM-MOTION

The stars thus far considered form but a part of the whole number between longitudes 217° and 360°. Small parts of the sky have been neglected altogether; everywhere we have kept above

[1] They were computed by the formula $r_p=57^\circ\!.3r/\mu$ in which r=probable error of the proper motion in either α or δ. These were assumed to be equal and I took for r the mean of the probable errors given in Boss's catalogue for the proper motions in α and δ. The derivation of the formula offers no difficulty.

certain limits of proper motion. Now that we have definitive elements, we shall take all of the stars into consideration. In the catalogue at the end of this paper the observed position angles p of the proper motions and the radial velocities ρ have all been compared with the values they would have had in case the stars followed the stream-motion exactly. We shall briefly examine the stars, whose motions seem not at all, or but indifferently, reconcilable with the stream-motion: first, those having centennial motions exceeding 1″6.

A. Galactic Latitude North.—The position angles are extremely well represented. There are only four stars among 137 for which the deviation exceeds 40°. Meanwhile, we have excluded only stars for which the deviation exceeds 60°. There were two in this case:

Boss No.	$p_o - p_c$	r_p	$(p_o - p_c) \sin\lambda$	
2605	+ 62°	±34°	+ 45°	(36)
4225	−169	6	−154	

The first is pretty satisfactorily explained by the exceptionally large uncertainty in the observed proper motion. Thus, there is really but one case which cannot be admitted to the group on account of the direction of its motion.

In radial velocity the stars given in Table XX show divergences of over 10 km.

TABLE XX

Boss No.	Sp.	ρ_o	ρ_c	O−C
2428	B5	+32.0 (1 obs.)	+11.9	+20.1
3115	B9	−20.0 (1 obs.)	+12.6	−32.6
3176	B3	+28.0:	+14.3	+13.7
3187	B3	+25.0:	+14.8	+10.2
3699	B3	−16.6	+ 8.1	−24.7
3892	B8	+21.0 (1 obs.)	+ 4.9	+16.1
4190	B1p	−21.0 (1 obs.)	+ 4.9	−25.9
4287	B1l	−27.0	+ 3.0	−30.0

All the velocities have been taken from Campbell's catalogue of B stars. Six stars either have but one observation or are marked as uncertain. They will probably turn out to be spectroscopic binaries. No. 4287 also deviates largely in position angle, and must

certainly be excluded. No. 3699 has been retained in the expectation that, though not marked in any way in Campbell's catalogue, it may still turn out to be a binary. In all, therefore, the three stars Boss 2605, 4225, and 4287 have been excluded from the northern stars with centennial proper motions exceeding 1."6. One of them may still turn out to be a member of the group.

B. Galactic Latitude South.—The following 13 stars show divergences in position angle of over 60° and were for that reason excluded: Boss 2060, 2109, 2165, 1558, 1469, 1566, 4392, 4796, 4599, 4913, 4739, 4783, 4954.[1] For the first six stars the great divergence is fairly explained by the vicinity of the antivertex. One of them, No. 2165, may prove to be a member of the second stream. Whether this be so or not will be settled as soon as the radial velocity is determined. There thus remain seven exceptional stars in a total of 115. These are all within the limits 296° to 360° of galactic longitude, −9° to −30° of latitude. We could hardly expect otherwise than that near the limits of the group there should be some overlapping of other groups.

In radial velocity we have diverging over 10 km the stars named in Table XXI.

TABLE XXI

Boss No.	Sp	ρ_o	ρ_c	O−C	l
4796......	B9	− 4.2	+5.9	−10.1	310°
4429......	B3	+17.0: S.B. est.	+1.1	+15.9	319
4498......	B8	−16.0	−0.7	−15.3	325
4637......	B8	−17.0 1 obs.:	+0.5	−17.5	325
4999......	B9	−17.0 S.B. est.	−2.6	−14.4	342

The first and the last also have abnormal deviations in p ($p_o - p_c$ = respectively, −122° and −57°). As furthermore they are within the limits for which we found the exceptional position angles, there seems to be little doubt that these two stars do not belong to our group. Of the rest, two are marked as uncertain. Considering the probability that the binary character of many stars, especially B stars, must have escaped detection, I have

[1] For Boss 1702 the great divergence is no reason for exclusion. It is only 3° distant from antivertex.

retained them and also No. 4498, which has no special uncertainty mark.

Summing up, we have excluded, of the stars with proper motions exceeding 1".6, 17 in a total of 252. It is not improbable that seven of these really belong to the group. Their proximity to the antivertex makes a decision as yet impossible. Thus there may be not more than 4 per cent of the whole number of the stars which do not belong to the group.

II. STARS WITH SECULAR PROPER MOTIONS $\leqq 1''.6$

In all there are within the area here under consideration 51 stars with this small proper motion. The direction of the motion is of course generally very uncertain. In order to decide whether they belong to the group, I followed two methods:

1. There are 21 stars—40 per cent of the whole number—for which the probable error of the proper motion is less than half the motion. For these I find the values of $p_o - p_c$ and $(p_o - p_c)\sin\lambda$ given in Table XXII.

TABLE XXII

p_o-p_c	$(p_o-p_c)\sin\lambda$	p_o-p_c	$(p_o-p_c)\sin\lambda$	p_o-p_c	$(p_o-p_c)\sin\lambda$
−39°	− 9°	−44°	−13°	−38°	−38°
+25	+ 4	−27	−11	+30	+27
−67	−27	+ 9	+ 5	−42	−38
−45	−14	+75	+43	− 3	− 3
+57	+20	− 5	− 3	+28	+25
−60	−25	+ 5	+ 5	−13	−12
− 1	0	− 2	− 2	−15	− 9

The mean absolute value of $(p_o - p_c)\sin\lambda$ is only 16°. This astonishingly low value leads, in my opinion, to a strong presumption of membership in the group. At the same time it furnishes a splendid testimony as to the reliability of Boss's proper motions.

2. If we exclude the values of the radial velocities (see List 3) depending on but one observation, and those marked as uncertain, there remain in all 9 values, which deviate from the computed values by the following amounts in kilometers: −3.3, −3.1, −1.3, +15.0, +8.1, −3.3, −15.5, +5.9, −7.9. Two, those of Boss 2467 and 5240, are widely divergent. The latter star is nearly at

the extreme limit of the group. The rest agree about as well as we should expect from the probable error (26) obtained before.

If we keep in mind the probability that even among the B stars for which three or more observations show no very marked divergences, there must still be numerous undetected spectroscopic binaries, we are led, I think, to the following conclusion: with the possible exception of a very few objects, the bright stars having centennial proper motions below 1."7 also belong to the group.

12. THE PARALLAXES AND THEIR PROBABLE ERRORS

As we are concerned with a group of stars, the members of which move so nearly parallel, we are naturally led to attempt to apply to them the method that has been used for finding the parallax of the members of such groups as the Hyades and Ursa Major.

The component of the linear motion of a star at right angles to the line of sight, in the plane containing the vertex is (see Fig. 3) $V \sin \lambda + u$ km per second. This is equivalent to $0.212\ (V \sin \lambda + u)$ solar distances per year. The angle under which we see this motion, that is, the component of the proper motion along the great circle toward the antivertex, is $v = \mu \cos \Delta p = \mu \cos (p_o - p_c)$. The corresponding linear motion is v/π, π being the parallax. Equating the two we get for the parallax

$$\pi = \frac{v}{0.212\ (V \sin \lambda + u)} \tag{37}$$

In our case, where $V = 18.3$ km (27), and the probable amount of u is ± 2.1 km (33), we may write without appreciable error, when $\sin \lambda$ is not too small,

$$\pi = \frac{v}{0.212\ V \sin \lambda}\left(1 - \frac{u}{V \sin \lambda}\right)$$

As, in the case of numerous stars, the values of u were assumed to be distributed according to the same law as accidental errors (see Section 9), this is equivalent to saying that

$$\pi = \frac{v}{0.212\ V \sin \lambda} \tag{38}$$

with a divergence whose probable amount is

$$r'_\pi = \frac{r_u}{V \sin \lambda}\,\pi \tag{39}$$

These formulae prove that we can thus gain considerable insight into the arrangement of the helium stars in space. With the values of V and r_u just quoted we find, for instance, for the stars 90° from the vertex, $r_\pi = 0.115\pi$ which, in the present state of science, must be considered a very high precision indeed.

Meanwhile, the value (39) does not represent the total probable error of the parallaxes. We must add the effect of the observation errors on the quantities v. This effect will be small in the case of large proper motions. It becomes very sensible in the case of many of the helium stars, which in general have small proper motions.

v being a component of the astronomical proper motion, its probable error must be about the same as that of any other component of this motion. I shall therefore take for it the mean, r_o, of the probable errors of the proper motion in right ascension and declination which are given in Boss's catalogue. We thus have P.E. of π due to observation error in $v = \frac{r_o}{0.212\, V \sin\lambda} = \frac{r_o}{\mu \cos(p_o - p_c)}\pi$

Therefore, finally (r_π being the total P.E. of π),

$$r_\pi = \pi \sqrt{\left(\frac{r_u}{V \sin\lambda}\right)^2 + \left(\frac{r_o}{\mu\cos(p_o - p_c)}\right)^2} \qquad (40)$$

With such excellent data as those of the Boss catalogue, the increase in the probable error due to observation error will be found very small, especially in the Northern Hemisphere, as long as the total proper motion μ is not below a few hundredths of a second.

13. APPLICATION TO THE HELIUM STARS

In the case of the helium stars we find, introducing the values (27) and (33),

$$\pi = \frac{v}{3.88 \sin\lambda} = \frac{\mu\cos(p_o - p_c)}{3.88 \sin\lambda} \qquad (41)$$

$$r_\pi = \pi \sqrt{\left(\frac{0.115}{\sin\lambda}\right)^2 + \left(\frac{r_o}{\mu\cos(p_o - p_c)}\right)^2} \qquad (42)[1]$$

[1] Owing to a mistake the computations for our catalogue have been carried through with the value $r_\pi = \pm 2.3$ instead of ± 2.1, so that instead of (42) we used

$$r_\pi = \pi \sqrt{\left(\frac{0.126}{\sin\lambda}\right)^2 + \left(\frac{r_o}{\mu\cos(p_o - p_c)}\right)^2}$$

As the consequence is only a slight increase in the probable errors of the values of π, I have not thought it worth while to repeat the computations.

With the aid of these formulae I have computed, accurate to 1 or 2 per cent of the amount, the parallaxes and their probable errors for all the stars with the exceptions of: (*a*) those which probably do not belong to the group (Vela stars and excluded stars); (*b*) those lying only a few degrees from the anti-vertex; (*c*) those having total centennial proper motions below 1.″0.

The computed parallaxes have been inserted in Lists 1, 2, 3, under the head π_1. The meaning of π_2 will be explained in Section 17. That of the other columns will be understood from the headings and the signification of the letters given at the end of this paper. For r_p see footnote on p. 71. The following summary will show at a glance the nature of the results obtained.

Of all the stars having centennial motions exceeding 1.″6 the percentages given in Table XXIII have parallaxes with probable errors within the limits given in the first column.

TABLE XXIII

Limits of P.E.	Galactic Latitude		Total 236 Stars
	North, 135 Stars	South, 101 Stars	
$r_\pi < \pi/6$	23 per cent	6 per cent	15 per cent
$\pi/6 < r_\pi < \pi/5$	33	7	21
$\pi/5 < r_\pi < \pi/4$	23	15	20
$\pi/4 < r_\pi < \pi/3$	12	24	17
$\pi/3 < r_\pi < \pi/2$	7	28	16
$r_\pi > \pi/2$	2	20	10
	100	100	99

For the stars north of the Milky Way the results appear to be very satisfactory. In 79 per cent of the cases the probable error is smaller than a fourth part of the parallax. For the stars south of the Galaxy they are not so good, owing mainly to the fact that so many of the stars are rather near the antivertex. For 38 per cent of these southern stars sin λ is below 0.60. In judging of the applicability of the method we have further to bear in mind that the entire group is south of the equator, in consequence whereof the accuracy of the proper motions—and of the parallaxes—is far below what would have been obtained had the group been in the Northern Hemisphere.

In consideration of the great importance which the knowledge of parallaxes has, it is natural to inquire as to what can be obtained in a similar way for the individual stars of the other spectral classes. It is true that among these we do not find any such great parallelism and equality of motion as among the B stars and before the discovery of the phenomenon of star-streaming it might well have seemed hopeless to apply to them the method here used for the helium stars.

Now, however, matters may turn out differently, at least in so far as we succeed in separating the members of the two streams. In each of them an approach to parallelism and equality of motion, though not so marked as for the helium stars, is very noticeable. Moreover, there are circumstances, such as the generally larger proper motion, the greater stream-velocity, at least of the first stream, etc., which compensate to a certain extent the influence of the greater inequality of motion. It is for these reasons that a separate section is devoted to a somewhat closer investigation of the matter (see Section 18).

14. MAP SHOWING THE ARRANGEMENT IN SPACE

With the aid of the parallaxes π_1 in the catalogue I have constructed a map showing the position in space of the stars treated in this paper (Map 3, p. 50). The positions shown are the projections on the plane of the Milky Way. The sun is supposed to be at the center. The polar co-ordinates therefore are: $r = \cos b/10\pi$ and l, b and l representing the galactic latitude and longitude. The stars in the Northern Hemisphere are shown as black disks, those of the Southern as circles. The differences in absolute magnitude are indicated by differences in the diameters.[1] The scale is given on the map. The very uncertain positions are marked with an interrogation point.

In using this map we must, of course, not forget that as we get farther away from the sun we lose more and more the absolutely faint stars. This is a consequence of the fact that our stars are complete only to what in Boss's catalogue is the sixth apparent magnitude (=5.80 Harvard). If we except the stars, few in

[1] For this computation, see next section.

number, which in this catalogue are somewhat fainter than the limit, we find on the map at $\pi=0''.0174$ only stars $+2.0$ or brighter; at $\pi=0''.0069$ only stars 0.0 or brighter; at $\pi=0''.0028$ only stars -2.0 or brighter, etc. For this and other evident reasons the map gives an idea of the relative star-density within each zone of constant distance but not—at least not without some computation—of the relative density in the consecutive zones.

I shall not indulge in speculations about the arrangement of the stars as shown by the map. It will be safer to wait until we shall have a similar representation for the whole of the bright helium stars near the Galaxy. I shall only draw attention to the pretty strong condensations at·

Gal. Long.		Gal. Lat.	Parallax	
220° to	240°	Mostly South	0''.0200 to	0''.0300
260	272	North and South	.0130	.0170
290	325	Mostly North	0.0090	0.0140

Of course we may see in the arrangement of these condensations the indication of a spiral structure. I shall not lay much stress on this, however, unless we find the same thing repeated in other parts of the sky.

15. ABSOLUTE MAGNITUDES AND THE LUMINOSITY-CURVE

The parallaxes being known, we can now find the absolute magnitudes. As in *Publication*, No. 11, of the Groningen Laboratory I will define the absolute magnitude of a star as the apparent magnitude it would show if placed at a distance corresponding to a parallax of $0''.1$. It is easily seen that if m = apparent magnitude, M = absolute magnitude, we have

$$M=m+5+5\log\pi \tag{43}$$

By means of this formula the values of M in the catalogue at the end of this paper have been computed. These I will now use for the derivation of the luminosity-curve. I will confine myself to the B0–B5 stars. There is evidently a rather large difference in luminosity between these and the B8–B9 stars, so that the two classes ought not to be combined. For the latter class by itself, however, the material seems too unsatisfactory. We shall have to wait for further data, or, perhaps, combine the B8–B9

stars with the A0–A3 stars. A provisional curve for the B5–B9 stars needed for a special purpose will be given farther on. See Table XXVI. As for the B0–B5 stars, let them be arranged according to both absolute magnitude and parallax. The process will be much simplified if we combine the stars of equal brightness within half a magnitude. We shall thus group the stars of absolute magnitude $+1.75$ to $+2.25$; those of magnitude $+1.25$ to $+1.75$, etc. Similarly we combine the stars within limits of parallax such that if a star were removed from the one limit to the other it would change its apparent brightness by just half a magnitude. We thus find Table XXIV.

The several columns are limited at such points that any star apparently fainter than 5.805 would necessarily find its place outside these limits. Some stars between apparent magnitudes 5.305 and 5.805 will also fall outside these limits, and are thus lost for our determination. This cannot be helped unless we take the intervals of magnitude, and correspondingly those of the parallaxes, smaller. The nature of the data does not seem to call for such refinement.

By the arrangement we are sure that within the limits of our table we have all the stars which exist, a condition essential for our purpose. Leaving out of consideration for the moment the numbers within parentheses, the construction of the table from the data in the catalogue will be understood if we add: first, that for stars precisely on or very near to the limit of two consecutive lines or columns, half a star was registered in the one and half in the other; second, the stars for which the centennial proper motion is $1''.6$ or smaller were kept separate from the rest by inserting them as a second number with the sign $+$. Take for instance the stars for $\log \pi = -2.361$ (mean of -2.411 and -2.311) and $M = -1.5$ (mean of -1.75 and -1.25). We find $2+1$, which means: two stars having $100\,\mu \geqq 1''.7$ and one having $100\,\mu \leqq 1''.6$. Third, the stars within 37° from the vertex or antivertex were excluded. For these the parallax, consequently the absolute magnitude, is more uncertain than for stars in other parts of the sky.

It is at once seen that this table furnishes a determination of the relative numbers of stars of different absolute magnitudes, that is, of the luminosity-curve. In fact, the numbers in each of

TABLE XXIV

Number of Stars

M	Log π: I −1.711 to −1.611	II −1.811 to −1.711	III −1.911 to −1.811	IV −2.011 to −1.911	V −2.111 to −2.011	VI −2.211 to −2.111	VII −2.311 to −2.211	VIII −2.411 to −2.311	IX −2.511 to −2.411	X −2.611 to −2.511	Total: Obs.	Comp.	O−C
+1.75 to +2.25	2 (0.6)										2	0.6	+1.4
+1.25 to +1.75	(0.7)	2 (2.4)									2	3.0	−1.0
+0.75 to +1.25	1 (0.7)	5.5 (2.6)	1.5 (4.0)								8	7.2	+0.8
+0.25 to +0.75	(0.7)	3 (2.5)	7 (4.0)	1.5 (6.4)							11.5	13.5	−2.0
−0.25 to +0.25	0.5 (0.6)	1 (2.3)	6 (3.6)	8 (5.7)	7 (8.7)						22.5	20.8	+1.7
−0.75 to −0.25	1.5 (0.5)	(1.9)	1 (2.9)	4.5 (4.8)	5.5 (7.2)	3.5 (5.8)					16	23.0	−7.0
−1.25 to −0.75	(0.4)	1 (1.4)	1 (2.2)	4 (3.7)	5 (5.5)	5+2 (4.4)	5+0.5 (3.5)				23.5	21.1	+2.4
−1.75 to −1.25	(0.3)	1 (1.0)	2 (1.6)	2 (2.8)	3 (3.8)	4 (3.1)	1+1 (2.5)	2+1 (2.1)			17	17.2	−0.2
−2.25 to −1.75	(0.2)	1.5 (0.6)	1 (1.0)	6 (1.7)	4 (2.5)	1 (2.0)	(1.6)	1+1 (1.3)	0+6 (4.5)		21.5	15.4	+6.1
−2.75 to −2.25	(0.1)	0.5 (0.4)	1 (0.6)	(1.0)	5 (1.5)	1 (1.2)	0+2 (0.9)	(0.8)	0+2 (2.7)	0+1 (1.5)	12.5	10.7	+1.8
−3.25 to −2.75	(0.1)	(0.2)	(0.3)	(0.5)	(0.8)	(0.6)	(0.5)	(0.4)	(1.5)	0+2 (0.8)	2	5.7	−3.7
−3.75 to −3.25	(0.1)	(0.1)	(0.2)	0.5 (0.3)	1 (0.4)	1 (0.3)	(0.3)	(0.2)	0+1 (0.7)	(0.4)	3.5	3.0	+0.5
−4.25 to −5.75	....	(0.1)	(0.1)	0.5 (0.1)	(0.2)	(0.1)	(0.1)	(0.1)	0+1 (0.3)	(0.2)	1.5	1.3	+0.2
Brighter .	....	....	....	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.2)	(0.1)	0	0.8	−0.8
Totals	5	15.5	20.5	27	30.5	17.5	9.5	5	10	3	143.5	143.3	

Domain of the stars apparently fainter than 5.80 (Harvard) and part of the stars 5.31 to 5.80

the vertical columns show the curve. The extent of the curve shown by the several columns—that is, by the stars in the several shells at different distances from the sun—is different, but as the part of the curve common to two consecutive shells is generally considerable, nothing is easier than to reduce the numbers in all the shells to what they would have been had the stars all been in one and the same shell. It is evident that we must not combine the numbers without such a reduction (as has been done by some astronomers) because the volume and the star-density in the consecutive shells is or may be very different. Executing the combination in the way which insures the greatest weight to the results, I find the curve[1] shown in Table XXV.

TABLE XXV

LUMINOSITY-CURVE B0–B5 STARS

M	Number	n	Formula (44)	O−C
+1.5	61	3	61.7	-0^5
+1.0	74	8	65.8	+8
+0.5	58	11^5	64.4	-6^5
0.0	66	22^5	57.9	+8
−0.5	34	16	48.0	−14
−1.0	42^5	23^5	36.5	+6
−1.5	27	17	25.5	$+1^5$
−2.0	25^5	21^5	16.4	+9
−2.5	12^5	12^5	9.7	+3
−3.0	2	2	5.3	−3
−3.5	3^5	3^5	2.7	+1
−4.0	1^5	1^5	1.2	0

I omitted the number for $M=+2.0$, which is too uncertain. The column n shows the total numbers of stars that have contributed to each separate result. They are meant to give a rough idea of the relative reliability of the numbers in the preceding column, which must be considered as representing the observed luminosity-curve.

I have tried to represent these numbers by the analytical expression derived for the luminosity-curve of all the spectral classes together,[2] which Schwarzschild[3] found to be consistent with the

[1] The two stars for $M=+2.0$, log $\pi=-1.661$, were first equally divided over the two absolute magnitudes $+2.0$ and $+1.5$.

[2] *Groningen Publication*, No. 11, 1902.

[3] *Astronomische Nachrichten*, **190**, 361, 1912.

other empirically derived frequency-curves, determining, of course, the constants so as to represent the observations. I found

$$\log \text{Number} = 1.763 + 0.1285\, M - 0.0726\, M^2 \tag{44}$$

by which the last column but one of Table XXV was computed. The agreement seems as good as could have been expected. We may write this formula in the form (taking the total number of stars as our unit)

$$\text{Frequency} = \frac{h}{\sqrt{\pi}} e^{-h^2(M-K)^2} \tag{45}$$

in which

$$h = 0.409, \qquad K = 0.885 \tag{46}$$

The luminosity-curve of the B0–B5 stars is thus found to be a Gaussian error-curve. The absolute magnitudes deviate from the value K as do observed quantities in the case of a probable error of 1.17 magnitudes.

This luminosity-curve is to be considered as an observed curve only as far, approximately, as the maximum at $M = +0.9$. The intensely interesting question arises: What will be the curve for fainter absolute magnitudes? Will it still follow formula (44) or (45)?

In order to settle this question satisfactorily we shall necessarily have to wait for data for the stars having apparent magnitudes fainter than 6.0, especially of those of great proper motion. For several years such data have been collected at Groningen. Unfortunately the spectral classification of these stars is still wanting.

In the absence of such data nothing really definitive can be maintained. Still we are not altogether without an indication. According to Miss Maury's classification of the spectra of the Pleiades, we have in this group not a single star of the class B0–B4. Of the B5–B9 stars she finds the numbers in the third column of the summary given in Table XXVI.

The faintest star classified as B is 6.61 (Hertzsprung). The stars fainter than this are of the type A, those fainter than 9.5 mostly F (Tickhoff). As the stars are practically all at the same distance, the apparent magnitudes must differ from the absolute values only by a constant. The numbers of stars given for the

different apparent magnitudes thus show: (*a*) That, beginning at the brighter extremity the luminosity-curve of the B5–B9 stars first rises to a maximum and then falls off again to zero. Notwithstanding the small numbers of stars this point must be considered as well established. (*b*) That the curve is roughly symmetrical with respect to the maximum. Meanwhile, it cannot be taken as at all certain a priori that the luminosity-curve in such a local group, in which all the stars probably started on their career at the same time, is identical with that of the stars in the rest of the sky. Whether there is such identity or not ought to be settled by observation. A group like the Pleiades, if we knew its parallax, might also throw some light on the question.

TABLE XXVI

LUMINOSITY-CURVE B5–B9 STARS

m	M	Number	Lum.-Curve
≦2.0	−1.7	....	0.2
2.5	−1.2	....	0.1
3.0	−0.7	....	0.3
3.5	−0.2	1	0.5
4.0	+0.3	2	0.9
4.5	+0.8	3	1.3
5.0	+1.3	0	1.6
5.5	+1.8	3	1.8
6.0	+2.3	2	1.8
6.5	+2.8	2	1.6
7.0	+3.3	....	1.2
7.5	+3.8	....	0.8
8.0	+4.3	....	0.5
8.5	+4.8	....	0.2
≧9.0	+5.3	....	0.2
Totals	....	13	13.0

In a communication to the Academy of Sciences of Amsterdam (February meeting, 1912) I made an estimate of this parallax, founded on the hypothesis that the motion of the group in space referred to the center of gravity of the whole stellar system is parallel to the plane of the Milky Way. We have already had occasion to refer to the probability of such parallelism in the last part of Section 4. This hypothesis led to the parallax

$$\pi(\text{Pleiades}) = 0''.018 \qquad (47)$$

With the aid of this value the apparent magnitudes in Table XXVI were converted into absolute magnitudes, which are shown in the second column. On the other hand, I derived, as well as the material contained in the present paper permits—which, as was said just now, is not very well—the luminosity-curve of the B5–B9 stars. I find for it the equation (45) in which

$$h=0.508, \quad K=2.00 \tag{48}$$

If, with these values, we compute the distribution of the total of 13 stars over the different absolute magnitudes, we find the last column of Table XXVI. Considering the circumstances, the agreement between the numbers of this column and the observed numbers is tolerable. It seems by no means impossible that among the stars fainter than 7.0, classified as A stars, there are two or three which ought really to have been classified as B8 or B9. If this were so, the agreement would leave nothing to be desired.

How all this bears on our question is evident. We have, however, to bear in mind that the luminosity-curve defined by the constants (48) is again to be considered as given by observation only up to a certain point, in this case about $M=+2.0$. Still a maximum near this magnitude is clearly indicated. In the Pleiades the maximum must lie somewhere near $M=+1.3$ or $+1.4$. The difference is easily accounted for by the uncertainties in the two determinations and in the parallax. It would even disappear altogether, and at the same time the range of magnitude from the brighter stars up to the maximum would become nearly identical, if it should turn out that two or three stars fainter than 6.6 in the Pleiades are really B8 or B9. All this leads to the belief that the absolute brightness of the Pleiades must be fairly normal, that is, about equal to that of other stars of the same spectrum in other parts of the sky.

This being granted, we may apply what we find in the Pleiades to the stellar system at large. Therefore, the foregoing conclusions (*a*) and (*b*) would hold for the whole sky. They are directly obtained only for the stars B5–B9, but it seems very unlikely that the stars B0–B5 should follow a very different law.

16. THE STAR-DENSITY

The number of Bo–B5 stars per unit of volume, at different distances from the sun, is easily obtained from Table XXIV. If the stars of the first three columns are taken together and if we call N_0 the total number of stars in these three shells, N_4 that in shell IV, etc., we find Table XXVII.

TABLE XXVII

$\dfrac{\text{Stars } M \leqq +0.75 \text{ in I+II+III}}{\textit{Idem} \text{ in IV}}$	$= \dfrac{29}{27}$	Therefore $N_4 = 0.931\ N_0$
$\dfrac{\text{Stars } M \leqq +0.25 \text{ in IV}}{\textit{Idem} \text{ in V}}$	$= \dfrac{25.5}{30.5}$	" $N_5 = 1.196\ N_4$
$\dfrac{\text{Stars } M \leqq -0.25 \text{ in V}}{\textit{Idem} \text{ in VI}}$	$= \dfrac{23.5}{17.5}$	" $N_6 = 0.745\ N_5$
$\dfrac{\text{Stars } M \leqq -0.75 \text{ in VI}}{\textit{Idem} \text{ in VII}}$	$= \dfrac{14}{9.5}$	" $N_7 = 0.679\ N_6$
$\dfrac{\text{Stars } M \leqq -1.25 \text{ in VII}}{\textit{Idem} \text{ in VIII}}$	$= \dfrac{4}{5}$	" $N_8 = 1.25\ N_7$
$\dfrac{\text{Stars } M \leqq -1.75 \text{ in VIII}}{\textit{Idem} \text{ in IX}}$	$= \dfrac{2}{10}$	" $N_9 = 5.00\ N_8$
$\dfrac{\text{Stars } M \leqq -2.25 \text{ in IX}}{\textit{Idem} \text{ in X}}$	$= \dfrac{4}{3}$	" $N_{10} = 0.75\ N_9$

From these numbers, if we consider that we have approximately volume I+II+III = volume IV, and further that the volume of each following shell is almost exactly double that of the preceding one, we find Table XXVIII.

TABLE XXVIII

Limits of Distance		Densities	
0 to	8	$\Delta_0 = 1.00$	
8	10	$\Delta_4 = 0.93$	
10	13	$\Delta_5 = 0.56$	
13	16	$\Delta_6 = 0.21$	
16	20	$\Delta_7 = 0.07$	
20	26	$\Delta_8 = 0.04$	$\left.\right\}$ 0.06
26	32	$\Delta_9 = 0.11$	
32	41	$\Delta_{10} = 0.04$	

Of course these densities, particularly at distances beyond $20 (\pi = 0\overset{''}{.}005)$, are very uncertain. If, neglecting shell X, we combine VIII and IX, and compare this directly with I, II, and III, we get $\Delta_{8+9} = 0.06$. The absolute number of stars per unit of

volume of absolute magnitude $M=0$ is obtained by dividing the numbers in Table XXVIII by about 150. The numbers for the other absolute magnitudes are then obtained by the aid of the luminosity-curve (45) and (46).

The numbers of Table XXVIII show that we have to do with a group which thins out very rapidly as we go outward, that is, with a group pretty sharply limited in distance as well as in longitude and latitude. This conclusion is independent of the faintest stars. I mean that it ought not to be altered if later we should be able to take the stars fainter than 6.0 into consideration.[1]

Toward the greater longitudes (see Map 1) the collection of stars with which the present paper deals is limited at about 330°, though a number of outlying objects beyond this limit are included. Toward the north and the south the group cannot extend much beyond ±30° of galactic latitude. We have not discussed the parts of the sky beyond these latitudes, but it is easy to convince oneself that the helium stars thin out very rapidly there. Toward the south the thinning out begins in much lower latitudes. Toward the north such is the case below longitude 260°. In the smaller longitudes there is a pretty sharp limit at 217° (see Section 4).

In the direction opposite the sun it was found just now that the group has thinned out enormously at the distance corresponding to a parallax of $0\overset{''}{.}004$. Toward the sun (see Map 3) it can hardly extend much beyond $\pi=0\overset{''}{.}040$. In most longitudes it does not reach this limit. Outside these limits there must be, on all sides, an extensive space with hardly any helium stars. Those within the limits have a common motion relative to the sun which may be due wholly to the sun's own motion, from which the individual motions diverge very little—not much more, perhaps, than the individual stars in the Ursa group.[2] I think that we ought to call such a collection of stars a *physical group*. If exception is taken to the term, we must invent another with which to indicate the peculiar nature of the collection.

[1] It is true that we have here neglected the scattering of light in space. Within the distances with which we have to deal here the influence of such a scattering must certainly be small.

[2] Cf. Ludendorff, *Astronomische Nachrichten*, **183**, 113, 1909; **195**, 369, 1913.

17. SYSTEMATIC ERRORS IN BOSS'S CATALOGUE

Thus far we have not considered the possibility of any systematic error in the proper motions of the stars in Boss's catalogue. As in such a work the error must certainly be relatively small, there can scarcely be introduced any appreciable uncertainty by neglecting it altogether, if all the stars with which we have to deal have moderately large proper motions. In the present case, however, the proper motions are generally small. It thus becomes extremely desirable to consider whether means cannot be devised to correct our results for any remaining systematic error.

If we accept the result reached in Section 10, that practically all the helium stars in our regions, whatever their proper motions, form a single system, the derivation of such a correction is by no means hopeless. For it must be evident that the influence on the position angles, p, of a systematic error in the proper motions in right ascension ($\delta\mu_\alpha$) or in declination ($\delta\mu_\delta$) will be the more sensible the smaller the proper motion. Therefore, if such an error exists, we must find that for any point of the sky, though the direction of the motion of all the stars be really the same, this direction, as derived from Boss's data, will be different for stars of small and great proper motion. We shall have to accept as the systematic catalogue-correction one that will make the difference disappear.

Mathematically speaking, the problem is indeterminate. We obtain only one equation between the two unknown quantities $\delta\mu_\alpha$ and $\delta\mu_\delta$, but in many cases the circumstances of the case will in great part remove the indeterminateness. For stars with small proper motion, let μ_1 and p_1 represent the average proper motion and position angle. Let μ_2 and p_2 represent the same quantities for stars of larger motion in the same part of the sky. Neglecting quantities of the order of the squares of $\delta\mu_\alpha/\mu$ and $\delta\mu_\delta/\mu$, we find, in order that the two sets of stars may furnish the same value for the position angle, that the corrections $\delta\mu_\alpha$ and $\delta\mu_\delta$ must satisfy the equation

$$p_2-p_1=\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\cos p\cdot\delta\mu_\alpha-\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\sin p\cdot\delta\mu_\delta \qquad (49)$$

in which, to the order of our approximation, we may take for p any value sufficiently near p_1 and p_2. For the regions A, B, ... this equation gives the conditions in Table XXIX.

TABLE XXIX

$$A,\ -0.82\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\delta\mu_\alpha+0.57\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\delta\mu_\delta=\frac{p_2-p_1}{57.3}\ (96\text{ stars})$$

B, −0.99	"	+0.15	"	=	"	(24 ")
C, −0.37	"	+0.93	"	=	"	(28 ")
D, −0.45	"	+0.89	"	=	"	(20 ")
E, +0.38	"	+0.92	"	=	"	(25 ")
F, 0.00	"	+1.00	"	=	"	(20 ")
G, −0.27	"	+0.96	"	=	"	(11 ")

The stars having centennial motions below 1″0 are few in number and their position angles are of course exceedingly uncertain. Their consideration cannot help us appreciably, and we shall neglect them in what follows.

In the region A there are only two, and in B only seven stars, for which the total proper motion lies between 0″010 and 0″016, whose position angles have not at the same time probable errors exceeding 30°. The average for these nine stars was compared with that for the stars of greater proper motion. We have, of course, reduced to the same point of the sky. In order to make this reduction small we combined the normals with the respective weights 1 and 3. We thus found the two averages:

α	δ	μ	p	P.E.	
17^h5^m	−44°1	0″049	195°5	±3°	(50)
17 5	−44.1	0.0135	192.0	±6	

Substituting these values into the equation obtained by combining the first two of Table XXIX, also with the weights 1 and 3, we find

$$51\delta\mu_\alpha+14\delta\mu_\delta=+0''061 \qquad (51)$$

The difference in the values of p in (50) is far within the probable errors of the determination. Moreover, even if real, it will only necessitate a correction to the proper motion either of right ascension or of declination, by which the position angle of the normals for A and B in Table XVIII (and these have been used in the determination of the vertex) would be changed hardly more than one degree.

In region D there is not a single star with proper motion below $0''.017$ and r_p below $30°$. The remaining regions C, E, F, G are by far the most important for fixing the true position of the vertex. With the exception of a relatively small part, of which the declination exceeds $-60°$, the whole region is practically contained between the limits

$$\left.\begin{array}{l} 7^h0^m \text{ and } 12^h30^m \text{ in } \alpha \\ -30° \text{ and } -60° \text{ in } \delta \end{array}\right\} \quad (52)$$

We shall assume that the corrections $\delta\mu_\alpha$ and $\delta\mu_\delta$ are constant for this region. In order to avoid the trouble of reducing all the results to exactly the same point in the sky, I operated with the values of $p_o - p_c$, the values of p_c being computed for the vertex (27). The results are in Table XXX.

TABLE XXX

Limits of μ	$\bar{\mu}$	$p_o - p_c$	Probable Error	Smoothed	n	Weight
$0''.010$ to $0''.019$	$0''.015$	$-11°$	$\pm 7°.5$	$-2°.2$	13	2
0.020 0.025	0.023	-1.5	4.0	-0.8	11	7
0.026 0.029	0.027	$+10$	5.0	0.0	7	5
0.030 0.039	0.034	$+2$	4.5	$+1.1$	11	6
0.040 0.049	0.046	-2	2.5	$+3.1$	13	14
0.050 0.079	0.063	$+12$	4.5	$+6.0$	8	5
≥ 0.080	....	....	....	....	0	....

The weights have been roughly computed from the interagreement of the observations. There is evidently a gradual change in $p_o - p_c$ with $\bar{\mu}$. As the numbers run very irregularly I have smoothed them by assuming that they must satisfy the condition $p_o - p_c = a + b\mu$. By least squares we find $a = -4.7$ and $b = 169$. Combining now the first two smoothed values and similarly the last two, we find as the basis of further calculations

$$\left.\begin{array}{cc} \mu & p_o - p_c \\ 0''.019 & -1°.5 \\ 0.050 & +4.6 \end{array}\right\} \quad (53)$$

These values must now be substituted into the conditions of Table XXIX for the regions C, E, F, G, combined into one. Giving half-weight to the last, which rests on only 11 stars, I find

$$-1.3\delta\mu_\alpha + 31.0\delta\mu_\delta = +0.106 \quad (54)$$

The influence of a systematic correction in the proper motion in right ascension thus turns out to be quite negligible. No reasonable value assumed for it will change our conclusion as to the value of $\delta\mu_\delta$ or the position of the vertex. We may thus safely neglect it altogether and so find

$$\delta\mu_\delta = +0.''003 \tag{55}$$

It is worthy of remark that, according to (51), the same correction will also reduce practically to zero the small difference between the values for p found from the stars with great and small proper motions in the regions A and B.

Summarizing, we conclude: the systematic correction needed by the proper motions in right ascension for the regions A and B is so small that it cannot have any serious influence on the direction of the vertex furnished by the stars in these regions. For the remaining regions the influence of such a correction, even if somewhat more considerable, must be practically negligible (see co-efficients in Table XXIX). A systematic correction of the proper motions in declination will affect very little the corrections found for A and B. For the remaining regions there is evidence of such a correction to the amount of $+0.''003$. This correction will at the same time improve the agreement of the small and great proper motions in regions A and B.

If, therefore, we adopt for all of our regions the systematic corrections

$$\delta\mu_\alpha = 0.''000 \qquad \delta\mu_\delta = +0.''003 \tag{56}$$

we shall obtain the corrected values of p given in Table XXXI which must replace the normals of Table XVIII.

TABLE XXXI

Region	α	δ	100μ	Corrected p	n	(O−C) sin λ
A..........	$15^h\ 4^m$	−37°8	4.″3	217°0	96	0
B..........	17 45	−46.2	5.1	189.6	24	1.5
C..........	11 52	−50.4	4.4	251.8	28	3
D..........	12 43	−68.6	3.5	247.4	20	2.5
E..........	8 38	−60.2	3.3	297.3	25	2.3
F..........	8 27	−48.8	3.2	275.7	20	3
G..........	7 43	−38.5	3.0	262.6	22	0

These give for the corrected position of the vertex

$$18^h24^m, \quad +39° \tag{57}$$

The distances at which the several directions pass this point, read from the globe, are in the last column of Table XXXI. They are quite as satisfactory as those for the normals in Table XVIII. Adopting this new position we have also to repeat the computation of the stream-velocity from the radial motions. I now find

$$V = -18.0 \text{ km}, \quad K = -4.8 \text{ km} \tag{58}$$

The residuals left by this solution are shown in the last column O−C′ of Table XIX. They are also quite as good as those of solution (27). The correction $\delta\mu_\delta$ here found agrees at least in sign with the correction $-0\overset{''}{.}0023 \cos \alpha$ given by Boss himself.[1] For the region (52) this correction amounts on the average to $+0\overset{''}{.}0017$; for all of the regions, to $+0\overset{''}{.}0014$. Altogether I think that the present solution (57) and (58) is to be preferred, and I regret now that I did not base the further discussion on it. The changes introduced are, however, generally slight. Those in μ and p have not been given in the lists. They are easily computed by

$$\left.\begin{aligned} \delta\mu &= +0\overset{''}{.}003 \cos p \\ \delta p &= -\frac{0.17 \sin p}{\mu} \end{aligned}\right\} \tag{59}$$

Only the new parallaxes (π_2) are given. These will be found in the last column of the Lists 1, 2, 3. The values of M were also computed, but as they seldom differ more than 0.1 or 0.2 mag. I have omitted them.

18. POSSIBILITY OF APPLYING THE METHOD TO OTHER SPECTRAL CLASSES

In Section 13 we were led to the question whether results for parallaxes could not be obtained for the stars of other spectral classes in the same way as for the B stars. In order to obtain an insight into the matter I began by deriving provisional values for the stream-elements of the other classes, confining myself to the

[1] *Preliminary Gen. Cat.*, Introduction, p. xxviii.

stars belonging to the first stream. The second stream is decidedly less promising.

In what follows, therefore, the supposition is involved that we succeed in separating the two streams, that is, in determining to which of the two streams each star belongs. Now that the data for radial velocities are becoming more abundant, at least for the brighter stars, the doubtful cases will be confined more and more to narrow limits. The elements adopted are given in Table XXXII.

TABLE XXXII

	Vertex		V	P.E.	K	P.E.
A stars............	18^h22^m	$+18.8$	-27.2	± 1.3	0.0	± 0.8
F, G, K stars......	18 3	$+14.6$	-30.0	....	0.0	....

For the A stars they were taken from a still unpublished investigation by myself. The stream-velocity depends on the radial velocity of 70 stars, mostly obtained at Mount Wilson. For the F, G, K stars the vertex was taken from Eddington's paper in the *Monthly Notices* of November 1910. The stream-velocity used is somewhat smaller than would follow from Eddington's and Campbell's determinations.

With the aid of these elements I derived the values of r_π for the several classes. For the A stars I used the radial velocities which served for the determination of V, rejecting all the values depending on only one or two observations. For the F, G, K stars I used the velocities in *Lick Observatory Bulletin*, No. 229. I confined myself to stars having a secular proper motion exceeding $3''$, which does not deviate in position angle from the direction of the stream-motion (projected on the sphere) by more than $\pm 15°$.

TABLE XXXIII

	Average Radial Peculiar Motion	$r_u = 0.845 \times$ prec. col.	n
A stars.............	7.1 km	$\pm$ 6.0 km	44 stars
F "	10.6	$\pm$ 9.0	31 "
G "	10.7	$\pm$ 9.0	21 "
K "	12.8	$\pm$10.8	55 "

I further excluded all the stars for which, according to the Lick *Bulletin*, the observed velocity is uncertain (1 or 2 obs., S.B. est., values marked (:) or without decimal). I thus found, freeing the radial velocities from the sun's motion, Table XXXIII.

Two G stars and two K stars which show quite exceptional divergences (respectively +60.2, −41.2, +72.5, +67.5) were excluded. It would seem that the exclusion of four stars in a total of 151 is sufficiently justified by the probability of undiscovered orbital motion. Substituting these values into (39) we find

$$\left.\begin{array}{lll} \text{A stars} & r'_\pi \sin\lambda = \pm 0.221\pi & \\ \text{F} \quad " & \quad " \quad = \pm 0.300\pi & \multirow{3}{*}{$\left.\begin{array}{l}\\ \\ \\ \end{array}\right\} \pm 0.330\pi$} \\ \text{G} \quad " & \quad " \quad = \pm 0.300\pi & \\ \text{K} \quad " & \quad " \quad = \pm 0.360\pi & \end{array}\right\} \quad (60)$$

There is a second means of deriving the value r_u and consequently of r'_π. Eddington[1] finds for the first stream stars of spectrum A, F, G, K, M, $hV = 1.516$, where $h = 0.4769/r_u$, consequently

$$r_u = 0.314V \qquad (61)$$

Assuming that this result is valid for the aggregate of the F, G, K stars, we have by (39)

$$\text{F, G, K stars} \qquad r'_\pi \sin\lambda = \pm 0.314\pi \qquad (62)$$

which agrees well with the value for $F+G+K$ in (60). Taking $V = 30.0$ from Table XXXII, we find for the mean of the two determinations

$$r_u = \pm 9.6 \text{ km}, \qquad r'_\pi \sin\lambda = \pm 0.320\pi \qquad (63)$$

On account of the factor sin λ, the probable error r'_π becomes rapidly larger near the vertices. In order to obtain somewhat more precise notions, let us confine our attention for the moment to that part of the sky—almost 77 per cent of the whole—which is more than 40° from these vertices. The mean value of sin λ will be 0.891 and we shall have on an average

$$\left.\begin{array}{ll} \text{A stars} & r'_\pi = \pm 0.248\pi \\ \text{F, G, K,} \quad " & r'_\pi = \pm 0.360\pi \end{array}\right\} \quad (64)$$

To these values ought to be added the effect of the observation errors in v, which are very different for different stars and different

[1] *Op. cit.*, p. 35.

regions of the sky. We may, however, for the present neglect them, because their effect, which for the B stars in this paper is very material, will be much less for the stars of the later types for which both the parallaxes and the proper motions are as a rule much larger. In the Northern Hemisphere, at least, and for stars down to magnitude 6.0 this effect must be almost vanishing. In a catalogue like that of Boss's about half of the stars (not of the B type) will be included in the present discussion. For this half we shall have the probable errors (64).

I fear that in the judgment of many the precision implied by these probable errors will not be appreciated at its true value. Errors of 25 and 36 per cent of the whole may seem excessive. In a certain sense they are, but if we consider for a moment the present standpoint of the science, we may be led to mitigate this judgment. Direct parallax determination cannot deal at all with the bulk of the stars. It has to limit itself to those nearest our system. For the moment this is practically a restriction to the stars with large proper motion. Thus direct determinations can at most teach us something about the structure of only an infinitesimal part of the universe. But even for this part direct determinations do not give better results than we here find possible for the bulk of the stars for which we have the necessary data for proper motion. Take for instance the most extensive and valuable series of such determinations at present in existence. I find that at the Yale Observatory 178 stars, having a proper motion over half a second yearly, have been observed for parallax. Of these the number of parallaxes with a probable error

$$\left.\begin{array}{l}\text{below } 0.248\pi \text{ is } 36, \text{ or } 20 \text{ per cent of the whole}\\ \text{below } 0.360\pi \text{ is } 55, \text{ or } 31 \text{ per cent of the whole}\end{array}\right\} \quad (65)$$

That is, we find that 20 and 31 per cent of the Yale stars of large proper motion are better determined than the bulk of our stars; 80 and 69 per cent are not so well determined. If then the method yields results for the bulk of the stars not inferior to those furnished by direct parallax determinations for only a small fraction of them, there is reason to view the results (64) with less dissatisfaction.

But there is more. We can obtain results very materially better than those furnished by (38), though, as far as I can see, we cannot

say exactly how much better. The following reasoning will make this clear. The divergences of our parallaxes from their true values are due solely to the divergence of the individual linear motions of the stars from the stream-motion. In the case of stars having exactly the stream-motion the parallax found by (38) must be faultless.[1] Even in the case of divergences, no error will be introduced as long as the component of the linear *peculiar* motion in the stream-direction is zero ($u=0$). Therefore, the greatest errors must be looked for in the cases for which the divergence of the velocities in the stream-direction is greatest, that is, for considerable positive or negative values of u. In general, of course, we have no means of judging whether the linear motion u is considerable or not. We have only angular motion to guide us. Still we can distinguish a class of stars in which considerable positive values of u must be abnormally frequent, and another class in which there must be exclusively negative values of u which are considerable. In the first the parallaxes found by formula (38) must be predominatingly too great, in the second they must all be too small.

The first class is that of the stars for which the value of the angular motion v is great (great proper motion stars). The second is that of the stars for which the v-component of the angular proper motion is zero or negative. For the second class the proposition is at once evident. Our formula gives zero or negative parallaxes, which are of necessity too small. As to the first, it must be evident that, *ceteris paribus*, the stars nearest our sun will have the greatest angular motion. But it is also no less evident that, *ceteris paribus*, the stars having the greatest linear velocity will show the greatest angular motion. Therefore, among the great proper motion stars we must have, not only a predominance of near stars, but also—and this is the point at present—stars of extreme linear motion. In conclusion, therefore, our formula must give systematic deviations for the extreme values of v; (a) for the extreme positive values of v it must give the parallaxes preponderatingly too great, therefore too great on the average; (b) for the zero or negative values of v the parallax found is always too small.

[1] For the moment we neglect the observation errors in the proper motion.

It must be evident—and this leads to the point we are aiming at—that, if we succeed in getting rid of these systematic divergences without increasing the errors for the intermediate values of v, we shall diminish the average probable error.

It is easy to see how this can be done. We have only to find the average value of the parallax for all the stars of any given magnitude which have the same given value of v. Then if for each of these individual stars we adopt this mean value as its parallax, systematic error is evidently absolutely avoided.

The execution of this plan demands the knowledge: (*a*) of the star-density D at different distances from the sun; (*b*) of the luminosity-curve. Near the sun, up to $\pi = 0''02$ or $0''015$, I will assume the density D to be constant; for greater distances I will use the formula derived by Schwarzschild.[1] For the luminosity-curve I followed *Astronomical Journal*, No. 566, according to which

$$\left.\begin{array}{l}\text{Frequency abs. mag. } M \text{ to } M+\delta M = \dfrac{h}{\sqrt{\pi}} e^{-h^2(M-K)^2} \delta M \\ \qquad h = 0.248, \qquad K = 9.5\end{array}\right\} \quad (66)$$

The derivation of the necessary formula will be found in the appendix forming the last section of this paper. The computation, which is pretty laborious, was carried through for only the few cases inserted in Table XXXIV which will be sufficient for the present purpose.

TABLE XXXIV

VALUES OF $\bar{\pi}$ (FOR $V \sin\lambda = 27.0$ km)

Mag.	v					
	0''00	0''25	0''50	1''00	2''00	4''00
0.5	0''013	0''051	0''090	0''165	0''306	0''576
2.5	0.006	0.044	0.080	0.150	0.282	0.537
4.5	0.003	0.039	0.074	0.139	0.264	0.506
6.5	0.002	0.036	0.069	0.130	0.249	0.477

The computation was carried through for the value $V = 30$ km (Table XXXII) and $\sin\lambda = 0.90$. For other values of $\sin\lambda$, I multiplied the values of the table by $0.90/\sin\lambda$. This is not wholly accurate, but as $\sin\lambda$ is below 0.70 for only five of the

[1] *Astronomische Nachrichten*, **185**, 81, 1910.

stars, no serious error can have been introduced. As the table shows, the difficulty that to $v=0''.00$ correspond negative values of π has been removed. In order to see in how far the representation for the very great values of v has been improved we must make a comparison with the directly measured parallaxes.

TABLE XXXV

No. Gron. 24	Mag.	μ	v	π Obs.	r_π	π Computed		
						By (38)	By (d)	By (67)
304	0.3	0''.33	0''.33	+0''.056	±0''.036	0''.073	0''.060	0''.090
321	6.6	0.33	0.31	+0.077	0.035	0.069	0.044	0.052
280	0.1	0.35	0.33	+0.094	0.015	0.117	0.061	0.117
346	1.3	0.37	0.37	+0.138	0.014	0.059	0.065	0.074
148	6.1	0.42	0.41	−0.011	0.037	0.065	0.053	0.052
66	0.2	0.44	0.44	+0.066	0.020	0.078	0.085	0.094
139	5.9	0.44	0.43	+0.092	0.040	0.071	0.059	0.056
223	4.1	0.45	0.45	+0.066	0.021	0.093	0.081	0.073
310	7.0	0.48	0.09	+0.077	0.028	0.021	0.015	0.027
121	3.1	0.50	0.48	+0.061	0.036	0.079	0.070	0.074
164	2.2	0.51	0.51	+0.129	0.043	0.080	0.076	0.081
120	6.1	0.53	0.51	+0.085	0.028	0.093	0.073	0.063
252	6.3	0.53	0.13	+0.048	0.037	0.025	0.024	0.032
174	3.6	0.55	0.53	+0.058	0.015	0.084	0.074	0.074
224	6.8	0.56	0.49	+0.069	0.037	0.128	0.097	0.070
286	6.8	0.62	0.49	+0.021	0.018	0.124	0.094	0.069
162	6.3	0.63	0.60	+0.038	0.027	0.094	0.074	0.062
134	5.6	0.68	0.67	+0.071	0.020	0.112	0.090	0.072
130	5.5	0.75	0.72	+0.038	0.030	0.120	0.097	0.076
351	3.8	0.75	0.72	+0.041	0.044	0.117	0.099	0.085
247	6.7	0.83	0.67	+0.137	0.038	0.185	0.140	0.085
318	3.6	0.83	0.61	+0.097	0.037	0.117	0.115	0.087
339	6.9	0.84	0.81	+0.017	0.014	0.146	0.098	0.074
129	3.3	1.09	1.07	+0.092	0.009	0.172	0.143	0.108
14	5.8	1.34	0.98	+0.162	0.045	0.155	0.118	0.084
210	6.7	1.38	1.38	+0.133	0.031	0.301	0.217	0.109
6	4.3	2.07	1.64	+0.148	0.027	0.265	0.204	0.124
349	5.7	2.11	2.11	+0.157	0.008	0.353	0.243	0.128
7	2.9	2.24	2.22	+0.143	0.007	0.359	0.283	0.163
194	0.2	2.28	1.23	+0.075	0.006	0.236	0.221	0.165
36	5.9	2.31	1.11	+0.143	0.027	0.208	0.157	0.096
199	0.3	3.66	2.34	+0.759	0.010	(0.368)	(0.320)	(0.204)
17	5.3	3.75	3.75	+0.112	0.007	0.590	0.422	0.171
324	5.6	5.25	5.09	+0.311	0.004	1.112	0.571	0.230

Only the stars of considerable proper motion are worth considering. The measured values of the others are mostly illusory. Further, only those stars for which we have radial velocities can be assigned to the proper stream with some certainty. Of stars of known radial velocity having proper motions exceeding $0''.30$ yearly,

I found 59. Of these, 34 fit better in the first stream; 18 in the second, while for 7 it is quite uncertain to which stream they belong. The 34 assigned to the first stream will be found in Table XXXV. Some few diverge widely from the stream and in reality may not belong to it. As, however, we know a priori that among these stars of extreme proper motion we must look for extreme divergences, I have been careful to exclude no star as long as it clearly fits better in stream I than in stream II. The stars have been arranged in the order of the amount μ of the total proper motion.

The fifth column of Table XXXV shows the observed parallaxes, the sixth their probable errors. The seventh and eighth show the values computed respectively by the formulae (38) and (*d*) of the appendix (Table XXXIV). Taking the means of the first 11 stars, the 11 following, and the last 12, we get, giving equal weight to each star, Table XXXVI.

TABLE XXXVI

μ	$\bar{\mu}$	π Obs.	π Computed		
			By (38)	By (*d*)	By (67)
0.″30 to 0.″52	0.″420	+0.″077	+0.″073	+0.″061	+0.″072
0.52 0.83	0.660	+0.064	+0.109	+0.089	+0.070
$\geqq$0.84	2.360	+0.188	+0.355	+0.250	+0.138
Id. excl. 199	2.242	+0.136	+0.354	+0.243	+0.132

We clearly see that, according to expectation, formula (38) yields too high values for the very high values of μ. This becomes even more evident if we exclude No. 199 (α Centauri) which is very extreme and probably does not belong to the first stream. The agreement becomes considerably better by the new formula (*d*), but it cannot be denied that even this formula, for values of the proper motion exceeding 0.″84, gives values still too high. The cause may lie in part in the adoption of the value $V=30$ km. It is only in the first third of our table that A stars are included; for the rest all the stars are of the second type and for these the stream velocity, according to Eddington,[1] adopting Campbell's value 19.5 km for

[1] *Monthly Notices*, November 1910.

the sun's velocity, is 32.6 km. With this value the computed numbers for the stars with proper motions exceeding 0".52 must be diminished by 9 per cent.

Probably, however, the main cause of the divergence lies in the fact found by Schwarzschild[1] that the large deviations of u from the stream-motion are more frequent than they should be according to Maxwell's law, which has also been assumed in this paper. This must change our results in the desired direction. The same thing is proved by the radial velocities. Among 31 stars of the second type having proper motions exceeding 0".30, I find eight peculiar radial motions exceeding 29 km, and six exceeding 38.5 km. Assuming Maxwell's law and $r_u=9.6$ (63), these numbers would be respectively 1.3 and 0.2.

Schwarzschild's results are not directly applicable to our case and it will be necessary in a thorough discussion of the matter to include a reliable determination of the law of the velocities u. My assistant Zernike is at present working on the subject, using the radial velocities recently published by the Lick Observatory. This determination ought to be supplemented by a discussion of the astronomical proper motions of the stars of a single stream according to the method worked out by Schwarzschild. After this, the agreement with observation ought to leave little to be desired.

Not improbably we can still reduce the accidental divergences a little by the derivation of a formula giving π as a function of μ instead of v. In this way we return to the point of view taken in *Groningen Publication*, No. 8, with the following differences, however: (*a*) We have here considered a single stream and not all of the stars together. The errors of the individual parallaxes must thus be greatly reduced. In fact it is only this subdivision into two streams which makes the derivation of individual parallaxes promising. (*b*) We have a better insight into the probable errors of these individual parallaxes, at least of their upper limit. (*c*) The new formula will be theoretical, while that in *Groningen Publication*, No. 8, was derived empirically. Schwarzschild has already derived such a theoretical formula,[2] but this also applies to all of the stars in the two streams together.

[1] *Astronomische Nachrichten*, **190**, 361, 1912. [2] *Ibid.*, p. 373.

Meanwhile, the theoretical formula not yet being available, I have derived the following empirical equations for the representation of the observed parallaxes:

$$\log \pi = -0.866 - 0.036m + 0.505 \log \frac{v}{\sin \lambda} \tag{67}$$

$$\log \pi = -0.861 - 0.036m + 0.476 \log \mu \tag{68}$$

The values computed by (67) have been given in Tables XXXV and XXXVI. The coefficient for m was adopted in accordance with Schwarzschild's formula; the two others were derived from the data in Table XXXV, α Centauri being excluded It seems reasonable to expect that the definitive formula will represent these observed parallaxes nearly as well.

It is an unfortunate circumstance that the only direct way of ascertaining the degree of reliability of the parallaxes furnished by any formula, viz., that by direct comparison with the observed parallaxes, is so little satisfactory. That this is so, is not only a consequence of the relatively great uncertainty of the observed parallaxes, but also of the fact that the only really reliable observed values belong to stars of very large proper motion. Now these stars, as has already been remarked, must to a great extent be exceptional. There must be among them a high percentage of exceptionally large linear motions, therefore a high percentage of large divergences between theoretical and observed parallaxes. Necessarily we get by comparison with observations an exaggerated idea of the errors of the formula. We must get a better insight into these errors, by the earlier considerations of this section.

From another point of view, on the contrary, we have to consider the fact that exceptionally large errors must be most frequent for the large proper motion stars, as an advantage. For just these stars—those least amenable to a theoretical formula—have been and will in the future be preferred by parallax observers, so that for them too we shall get reliable data. It will only be necessary that in these direct determinations we give the greater care to the smaller parallaxes, in order that these too may become trustworthy within a moderate fraction of the total amount.

To sum up: formula (38) furnishes parallaxes for the individual stars, the probable errors of which are shown in (64). We can,

however, greatly reduce the larger deviations left by this formula if, instead, we use formula (*d*) of the appendix. Meanwhile it appears from the comparison with observations that even this formula leaves systematic errors for stars having a proper motion considerably over half a second yearly. The cause of this must lie in part in the fact that the value adopted for the stream-velocity of the second-type stars is too small, but for the greater part probably in a deviation of the distribution of the star-velocities from Maxwell's law. We must expect that formula (*d*) or another in which v is replaced by μ will systematically agree with observations as soon as we shall have a reliable determination of the velocity law for the members of a single stream. Such a formula will again reduce the errors of the theoretical value, but we must expect that even such an improved formula will leave rather large errors for the stars of exceptionally large proper motions. If, therefore, we leave the stars with proper motions of, say, $0\overset{''}{.}50$ or more for direct parallax determination, the probable error for the rest—the great bulk of the stars—will once more be lessened.

We may thus hope to find values for the parallaxes which will, perhaps, justify a first attempt to locate the stars in space. The attempt will, no doubt, be a very crude one. Still, it may lead to the disentanglement of some of the most salient points of the arrangement of the stars in space.

Meanwhile, before such an attempt can be made, we shall have to assign each of our stars to the stream to which it belongs. The astronomical proper motion being known, the possibility of this operation will depend in a great measure on the knowledge of the radial velocities. It thus becomes evident how urgent is our need for radial velocities of stars with well known proper motions, such, for instance, as those in the Boss catalogue.

It is rather probable that in the course of this preparatory work we shall find further means of reducing the probable errors of our theoretical parallaxes. The study of the helium and A stars led to the recognition of a certain number of local groups. The motion of these groups is appreciably different from that of the main stream though still in the main following either the one or the other. If it be true that the second type is only a later phase

in the evolution of the stars, then the expectation seems justified that among the second-type stars too we shall recognize partial streams. The knowledge of the radial velocities will be a great help in this discovery. Not only shall we find more decisive parallelism and equality of velocity in such partial groups, but the equality of motion of what is left of the main stream will be increased by the omission of such groups. Every increase in this equality, however, will mean decrease in the probable errors of our parallaxes. Even now the separate treatment of different parts of the sky may do much. It certainly does so in the case of the helium stars.

Finally, I wish to say that the method of the present article, embodied in equation (*d*) of the appendix, was not applied to the helium stars, in the first and most important place, because, owing to the fact that r_π is so much smaller for these stars, any marked increase in accuracy is not to be expected; in the second place, because the new formula is dependent on the law of the star-density and the luminosity-curve. Both these elements have been derived for the stars B0–B5, but not, or at least not satisfactorily, for the B8–B9 stars. Even for the former it might be well to wait for more extensive data.

19. PARALLAX DERIVED FROM APPARENT MAGNITUDE

In all that precedes, the finding of the parallax depends on an accurate knowledge of the astronomical and, for part of the material at least, of the radial motion. For the brighter stars, we have excellent data for the former element in Boss's already often-quoted catalogue. For the latter our data are rapidly increasing. For the fainter stars, on the contrary, what we know is not only extremely fragmentary, but it is evident that our further progress must be slow. As a consequence of this state of affairs, such investigations as were considered in what precedes, can extend our knowledge to but a small fraction of the whole universe. In our endeavor to conclude from this to the structure of the whole, we shall naturally try to get some help at least from other elements which may be obtained quickly for all or for a considerable part of all the stars we see in our telescopes.

Thus, for instance, the number of stars for each class of magnitude furnishes one of the most precious data. The magnitudes themselves have also been frequently used, though often in a way not unobjectionable. In what follows I shall attempt to turn them to account in a different way, starting from the supposition that we know the luminosity-curve.

For the sake of clearness and convenience, I shall here consider only the B0–B5 stars. The application to other spectral classes or to all of the stars together will present no difficulty as soon as for these too we know the luminosity-curve. First consider:

Problem I: Of a group of early B stars, all at practically the same distance from the sun, we have given the average apparent magnitude $\bar{m}$ *of all the members brighter than* m_0. *What is the parallax* π *of the group?*

As the frequency-curve has the form (45) and as

$$M = m + 5 + 5 \log \pi,$$

we get at once

$$\bar{m} = \frac{\int_{-\infty}^{m_0} m e^{-h^2(m+5-K+5\log\pi)^2} dm}{\int_{-\infty}^{m_0} e^{-h^2(m+5-K+5\log\pi)^2} dm} \tag{69}$$

which is readily transformed to

$$\bar{m} = K - 5 - 5 \log \pi - \frac{1}{2h} \frac{e^{-P^2}}{\int_{-\infty}^{P} e^{-z^2} dz} \tag{70}$$

in which

$$P = h(m_0 - K + 5 + 5 \log \pi) \tag{71}$$

Many tables have been given of the integral in the formula. In case π is known the value of $\bar{m}$ is readily computed. For the B0–B5 stars we have by (46) $h = 0.409$, $K = 0.885$. If further we take

$$m_0 = 5.80 \tag{72}$$

which is the limit on the Harvard scale to which Boss's catalogue is complete, we get the values of $\bar{m}$ in the second column of Table XXXVII.

The other columns will be explained presently. For the solution of our problem we have to enter this table with the argument

$\overline{m}$ and take out the corresponding value of π. Meanwhile, the applicability of the formulae is greatly restricted by the condition that all the stars must be practically at the same distance. Therefore,

Problem II: Of a group of early B stars, ranging over a wide interval of distance, given the average apparent magnitude of all the stars brighter than m_0; required the average parallax of the group.

TABLE XXXVII

π	$\overline{m}$	$\overline{me^{-1/8m}}$	Dev. from (76)
0".001	5.304	2.724	+0.005
	135	30	
.002	5.169	2.694	0
	114	26	
.003	5.055	2.668	− 1
	281	76	
.006	4.774	2.592	− 1
	247	76	
.009	4.527	2.516	− 1
	226	74	
.012	4.301	2.442	+ 1
	213	75	
.015	4.088	2.367	+ 2
	200	77	
.018	3.888	2.290	+ 1
	190	76	
.021	3.698	2.214	+ 1
	182	77	
.024	3.516	2.137	0
	169	77	
.027	3.347	2.060	− 1
	161	76	
.030	3.186	1.984	− 1
	484	248	
.040	2.702	1.736	+ 3
	413	239	
0.050	2.289	1.497	+ 17

Suppose we have a_1 stars of parallax π_1, a_2 of parallax π_2, etc. The preceding table furnishes the values of $\overline{m}_1$, $\overline{m}_2$, ... corresponding to these parallaxes. The total averages $\overline{\pi}$ and $\overline{m}$ will be

$$\left.\begin{aligned} \overline{\pi} &= \frac{a_1\pi_1 + a_2\pi_2 + \ldots\ldots}{a_1 + a_2 + \ldots\ldots} \\ \overline{m} &= \frac{a_1\overline{m}_1 + a_2\overline{m}_2 + \ldots\ldots}{a_1 + a_2 + \ldots\ldots} \end{aligned}\right\} \qquad (73)$$

We know nothing in general of the numbers $a_1, a_2, \ldots$. We have given only the average $\overline{m}$. If, entering Table XXXVII with the argument $\overline{m}$, we take out the corresponding value π_0 of the parallax, we shall certainly find π_0 different from $\overline{\pi}$. It is evident that this must generally be so in the evaluation of any quantity unless the tabulated value and the argument are linear functions of each other. In the present case $\overline{m}$ is not such a function, therefore π_0 must be different from $\overline{\pi}$.

For a moderate range of distance the difference $\pi_0 - \overline{\pi}$ will be small; still, as we usually know little or nothing about the range, it will generally be unsafe to take π_0 for $\overline{\pi}$. The difficulty may be avoided in the following way. In the preceding problem where we had to deal with stars of one and the same π, we expressed $\overline{m}$ as a function of π. It is this function which turns out to be non-linear. As, however, the individual magnitudes of all our stars are generally known, we may as easily express the average value of any function $\phi(m)$ of m as a function of π. We shall choose the form $\phi(m)$ in such a way that $\overline{\phi(m)}$ becomes a linear function of π.

Writing down the value of $\overline{\phi(m)}$ from analogy with equation (69), we thus have to determine $\phi(m)$ in such a way as to satisfy the equation:

$$\frac{\int_{-\infty}^{m_0} \phi(m) e^{-h^2(m+5+5\log\pi-K)^2} dm}{\int_{-\infty}^{m_0} e^{-h^2(m+5+5\log\pi-K)^2} dm} = A + B\pi \tag{74}$$

where A and B are constants.

I have not succeeded in finding the general solution of this integral equation; but as there is no real objection to taking for $\phi(m)$ any suitable form with any number of constants, it cannot be difficult to find an expression which will make the first member of (74) practically linear over a wide range of parallaxes.

For the present purpose I succeeded amply by the determination of a single constant, assuming for $\phi(m)$ the form

$$me^{-pm} \tag{75}$$

For $p = 1/8$ we find the values of the average amount of this function inserted in the third column of Table XXXVII.[1] The last column shows the divergences from the linear form

$$2.7445 - 25.3\,\pi \tag{76}$$

Between $\pi = 0''.001$ and $\pi = 0''.040$, which is a range wider than that of the parallaxes of the stars treated in this paper, these differences are practically vanishing. If, in the application to faint stars, even this range is deemed insufficient, we shall no doubt be able to extend it by the introduction of more constants.

In the special case of the bright B0–B5 stars we finally get the solution of our problem either by the use of Table XXXVII or by the formula obtained by adopting (76):

$$\overline{\pi} = 0''.1844 - 0''.0395\,\overline{me^{-1/8m}} \tag{77}$$

With a little table giving the values of the last term for values of m, the whole computation becomes extremely simple. I have applied this method to the B0–B5 stars in the several groups of this paper, entering the table first with the argument $\overline{m}$ and second with the argument $\overline{me^{1/8m}}$. The results are shown by Table XXXVIII.

The values in the fifth and sixth columns are practically identical in the present case. I have added the average parallax as found from the proper motion. They represent the best of what we know of these parallaxes. The agreement with the values derived from the magnitudes, which is perhaps best shown by the last column, may seem too rough to give them any claim to consideration.

In my opinion, however, their value is considerable. They show that some idea of average distance can be obtained from

[1] If we put $k = p/2h^2$

$$\gamma = 5 - k + 5\log\pi \qquad \theta = h(m_0 + \gamma + k)$$
$$\epsilon = h(m_0 + \gamma) \qquad g = 1/2p(k + 2\gamma)$$

we get

$$\overline{me^{-pm}} = -e^{g}\,\frac{(\gamma + k)\displaystyle\int_{-\infty}^{\theta} e^{-z^2}dz + \frac{1}{2h}e^{-\theta^2}}{\displaystyle\int_{-\infty}^{\epsilon} e^{-z^2}dz}$$

by which the average values of me^{-pm} for Table XXXVII have been computed.

the magnitudes alone, a precious knowledge in all those cases where we have as yet no data for the proper motions, or where that motion is so small that it becomes useless as a measure of distance.

TABLE XXXVIII

Region	Average Sp.	$\overline{m}$	$\overline{me^{-1/8m}}$	$\overline{\pi}$ from $\overline{m}$	$\overline{\pi}$ from $\overline{me^{-1/8m}}$	$\overline{\pi}$ from μ	n	$\log \frac{\text{col. 7}}{\text{col. 6}}$
				$100\mu \geqq 1''.7$				
A I	B3.0	4.26	2.468	0''.0126	0''.0110	0''.0081	19	−0.13
A II	B3.4	4.11	2.368	.0147	.0150	.0132	25	−0.06
A III	B2.8	4.20	2.443	.0134	.0120	.0089	28	−0.13
B	B2.9	3.97	2.317	.0168	.0169	.0112	14	−0.18
C+D	B3.3	3.98	2.337	.0166	.0162	.0129	23	−0.10
E+F+G	B3.3	4.59	2.547	.0082	.0078	.0173	24	+0.35
				$1''.0 \leqq 100\mu \leqq 1''.6$				
A+B+C+D	B3.5	4.93	2.637	.0043	.0042	.0033	11	−0.10
E+F+G	B3.0	4.88	2.642	0.0049	0.0040	0.0075	17	+0.27

The following examples, though still resting on inadequate data, will be sufficient as an illustration.

First example: Perseus cluster.—In a recent paper by Adams and Van Maanen[1] it has been shown that the stars belonging to the h and χ Persei cluster have a high radial velocity (−43 km), which enables us to distinguish surely between members and non-members of the cluster. Within the limits

$$\left.\begin{array}{ll} \alpha\ 1900 & 2^h8^m0^s \text{ to } 2^h17^m0^s \\ \delta\ 1900 & +56°20' \text{ to } +57°0' \end{array}\right\} \quad (78)$$

I find four B0–B5 stars, all members of the group, brighter than 7.20 (Harvard scale). Arranged in order of magnitude they are (spectra from *Astronomical Journal*, No. 648):

Harvard Mag.	Sp.	α 1900	δ 1900
6.40	B3	$2^h12^m12^s$	+56°42'
6.42	B2	9 52	+56 35
6.66	B4	12 3	+56 40
7.05	B2	9 47	+56 34

(79)

[1] *Astronomical Journal*, No. 648, p. 186, 1913.

The mean of the magnitudes is

$$\bar{m}=6.63 \tag{80}$$

Substituting

$$m_0=7.20, \qquad h=0.409, \qquad K=0.885, \tag{81}$$

the formulae (70) and (71) lead to

$$\left.\begin{array}{cc} \pi & \bar{m} \\ 0''.0005 & 6.71 \\ 0.0010 & 6.54 \\ 0.0020 & 6.37^5 \end{array}\right\} \tag{82}$$

Interpolation shows that to the observed value (80) of $\bar{m}$ corresponds the parallax

$$\pi=0''.0007 \tag{83}$$

which we conclude must be the parallax of the cluster. The uncertainty is of course still great.

We get an idea of the uncertainty by the following consideration. Given that on a certain area there are four stars between apparent magnitudes 6.30 and 7.20 and given the parallax, we can easily compute the number of stars brighter than 6.30 and the number of stars between, let us say, 7.20 and 9.0 to be expected within the same area. For with any given parallax we can at once transform our apparent magnitudes into absolute values, and the relative frequency of these is given by the luminosity-curve (45) and (46). I find the data given in Table XXXIX.

TABLE XXXIX

NUMBER OF STARS

π	Apparent Magnitude		
	$-\infty$ to 6.30	6.30 to 7.20	7.20 to 9.0
0''.004	4.3	4.0	9.5
.002	2.3	4.0	18.8
.001	1.3	4.0	38.8
0.0005	0.7	4.0	79.3

We conclude at once that the parallax cannot be as high as 0''.004 because in that case we ought to find within the area (78) at least four members of the cluster apparently brighter than 6.30

whereas there is certainly not a single one. Even the parallax 0."002 seems scarcely admissible for a similar reason. Therefore, even the extremely meager data available in the present case give us the right to conclude with some probability that the parallax of *h* and χ Persei must be below 0."002. To go farther would be dangerous. With such a small number of stars, the limits chosen for the area and for the apparent magnitudes are necessarily somewhat arbitrary. Moreover, relatively great accidental deviations from a normal distribution are to be feared.

Table XXXIX shows further how the observations of fainter stars will soon enable us to make a more accurate estimate. Even in the absence of radial velocities we shall be able to improve the result simply by the classification of the spectra for the fainter stars, for the number of such stars not belonging to the group on an area as small as that now under consideration must be very small and is capable of rough evaluation.

Considerations like these suggest the use of some such quantity as

$$\frac{\text{Number of stars apparently brighter than } m_1}{\textit{Idem} \text{ between } m_1 \text{ and } m_0}$$

instead of $\overline{m}$, for the derivation of π, I have briefly tried this modification of the method and think there are cases in which it may be preferable. The full development of the whole matter must, however, be delayed for a future publication.

Second example: Small Magellanic Cloud.—As far as I know there does not exist a complete catalogue of carefully determined magnitudes of all the stars in the small Magellanic Cloud. But there is an extensive list of variables in *Harvard Annals*, **60**, pp. 90–96. I assume that if we take the maximum light of these stars, we shall have a set of magnitudes that will be a fair specimen of all the magnitudes of the group. Fifty-four stars, well distributed over the cloud, mag. 14.0 or brighter—that is, on account of abbreviation to one decimal, brighter than 14.05—give

$$\overline{m} = 13.44 \tag{84}$$

From this datum I derived the parallax on two suppositions: (1) that we have to do with a set of stars similar in absolute magni-

tude to all the stars in the rest of the sky. As it may be urged that by going down only to magnitude 14.05 we include only the absolutely bright stars which therefore will belong exclusively or preferentially to the spectral types B or A, I supposed: (2) that we have to do with B0–B5 stars. The truth will probably lie between the two. We have for

I. "All the stars" $m_0=14.05$, $h=0.248$, $K=9.5$

II. B0–B5 stars $m_0=14.05$, $h=0.409$, $K=0.885$ Cf.(46)

The values of h and K for "all the stars" were taken from *Astronomical Journal*, No. 566, the formula there given being written in the form (45). By the aid of these values I find from (70) and (71) Table XL.

TABLE XL

π	$\bar{m}$	
	All Stars	B0–B5
0.″00005	13.43	13.38
.00010	13.36	13.16
.00015	13.32	12.97
.00020	13.27	12.78
.0004	13.15	12.15
.0008	13.01	11.15
.0016	12.82	9.83
.0032	12.56	8.36
.0064	12.20	6.85
.0128	11.72	5.35
0.0256	11.09	3.84

Interpolation, or rather extrapolation, for the value (84) thus gives

Parallax of Small Magellanic Cloud

$$\left.\begin{array}{ll} (1) & 0.''00004 \\ (2) & 0.00004 \end{array}\right\} \quad (85)$$

The result thus turns out to be independent of the spectral class of the stars. I have chosen the small Magellanic Cloud because another determination of its parallax, by a totally different method, has recently been made by Hertzsprung.[1] The value found—if we write out more decimals than was done by the author—is

$$\pi=0.''000084 \tag{86}$$

[1] *Astronomische Nachrichten*, **196**, 201, 1913.

The two results are at least of the same order of magnitude. As the value (84) of $\bar{m}$ which served for the derivation of (85) is professedly unreliable, Hertzsprung's result is probably the better of the two. But, as I possess a good photograph of the small cloud, I hope soon to secure the measures necessary to improve the present determination. Meanwhile, the determinations (83), (85), and (86), provisionally at least, must not be taken too seriously, for it has been tacitly assumed in all of them that there is no scattering or absorption of light in space. Owing to the enormous distance of the Magellanic Cloud the existence of any appreciable absorption will totally change the value derived for its parallax. Let a be the absorption in magnitudes per unit of distance ($\pi = 0''.1$), I then find that Hertzsprung's result will become

a	π	Abs. in Mags.	
0.000	0''.000084	0.00	
.005	.000223	2.24	
.010	.000318	3.14	(87)
.015	.000398	3.77	
0.020	0.000470	4.26	

Therefore, as long as the question of absorption in space is not settled, we can hardly maintain that we know anything of the parallax of this object.

Moreover, it must be acknowledged that even leaving alone the question of the existence of absorption of light in space, the matter is still in a somewhat unsatisfactory state. The deviations found a moment ago, especially those for the *E*, *F*, *G* group, both for the stars with greater and those with smaller proper motion, are somewhat exceptional and of the same sign. It would seem that in this region the B stars are absolutely somewhat fainter than the bulk of the early B stars. Unless such systematic deviations of the absolute brightness prove to be quite exceptional, very accurate data for distance cannot be expected from the method. Meanwhile it will be better to wait for further data and a more reliable luminosity-curve before entering on a more thorough discussion.

20. PARALLAX DERIVED FROM THE SPECTRUM OR THE COLOR INDEX

I think that at the present moment we can hardly refuse to admit that, apparent magnitude and spectral lines being the same, the color index of the stars farthest away is greatest. Corresponding to the change in color index, there must of course be a change in the spectrum, and the spectrum must yield an even more sensitive measure of the phenomenon than the color index. In a paper now ready for press[1] I have brought together whatever evidence, published and unpublished, I could collect.

Let us assume, as a first approximation, that this increase of color index is proportional to the distance, so that we may put

$$\text{Color index} = g + Cr \qquad (88)$$

The constant g, different for the several spectral classes, can easily be determined from the stars near our system. In fact, save for small corrections, it has already been determined by several astronomers. If now we suppose that by some means or other we have also found the value of C, it is evident that equation (88) will yield the value of the parallax as a function of the color index, which is directly measurable. The presence of the constant g restricts the application to stars of known spectrum. For these, however, it is not impossible that we shall obtain individual parallaxes. If so, then a very great field will be opened by the method. The question will have to be settled whether C is an absolute constant or whether it changes—and if so, in what manner—with the position in the sky, with the spectrum, and possibly with other circumstances.

The determination does not necessarily require a knowledge of the cause or causes to which the change of color index is due, however desirable such information in itself may be. If the form (88) proves insufficient, we may take another with more constants and thus proceed quite empirically. The method must become the more valuable, and proportionally the more precise, the farther away the objects are. If the phenomenon holds for the small Magellanic Cloud—which it does not necessarily—then the total

[1] *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, **40**, 1914.

effect cannot well be less, I think, than a full magnitude. It may be much greater.

On the whole, the small Magellanic Cloud may furnish an extremely valuable contribution to the whole question, in particular to the discrimination between possible causes and eventually to the determination of the constant C. The possession of three different methods for the derivation of its distance—that of Hertzsprung and the two indicated in this and the preceding section—is exceedingly valuable because of the mutual control and the increase in confidence with which the results of each will be regarded.

APPENDIX (TO SECTION 18). COMPUTATION OF THE AVERAGE VALUE $\bar{\pi}$ OF THE PARALLAX FOR STARS OF ANY GIVEN APPARENT MAGNITUDE AND A GIVEN VALUE OF v

Consider an infinitely small cone having its vertex in the sun, which cuts the area ω from a sphere drawn round the sun with radius unity. Take as unit of distance that corresponding to $\pi = 0\overset{''}{.}1$. The infinitely small volume dw cut from this cone by the two spherical surfaces round the sun, having the radii r and $r+dr$ will be

$$dw = \omega r^2 dr$$

Let

$\Delta_{m,r}$ = number of stars of *apparent* magnitude m per unit of volume at the distance r from the sun. This quantity must not be confounded with

D_r = number of stars of any arbitrarily chosen *absolute* magnitude, M per unit of volume at distance r.

Then

$$\omega r^2 \Delta_{m,r} dr$$

will be the number of stars of apparent magnitude m in dw. According to formula (28) of the text, the fraction

$$\frac{h'}{\sqrt{\pi}} e^{-h'^2 u^2} du$$

will have linear velocities between $V \sin \lambda + u$ and $V \sin \lambda + u + du$. For the value of h' we have by the aid of formula (63) of the text

$$h' = \frac{0.4769}{r_u} = \frac{0.4769}{9.6} = 0.050 \qquad (a)$$

The linear motion $V \sin \lambda + u$ is expressed in kilometers per second. Expressed in solar distances per year it becomes $0.212(V \sin \lambda + u)$. If v be the angle (in seconds) under which we see this motion, then

$$\frac{v}{\pi} = 0.212(V \sin \lambda + u)$$

from which, because

$$\pi = \frac{0''.1}{r} \tag{b}$$

$$u = 47.2 rv - V \sin \lambda \tag{c}$$

(u and V in kilometers per second; v in seconds per year; r in units defined by (b)). Therefore

$$\left.\begin{array}{l}\text{Number of stars in } dw\\ \text{having angular motion}\\ v \text{ between } v \text{ and } v+dv\end{array}\right\} = 47.2 \frac{h'\omega}{\sqrt{\pi}} e^{-h'^2(47.2rv - V \sin \lambda)^2} r^3 \Delta_{m,r} dr dv$$

and on account of (b)

$$\bar{\pi}_v = \frac{1}{10} \frac{\int_0^\infty e^{-h'^2(47.2rv - V \sin \lambda)^2} r^2 \Delta_{m,r} dr}{\int_0^\infty e^{-h'^2(47.2rv - V \sin \lambda)^2} r^3 \Delta_{m,r} dr} \tag{d}$$

It remains only to find $\Delta_{m,r}$. Let

$$\text{Frequency abs. mag. } M \text{ to } M + dM = \phi(M) dM \tag{e}$$

$\eta = \phi(M)$ represents what is usually called the luminosity-curve. As $M = m - 5 \log r$, we readily see that

$$\Delta_{m,r} = D_r \phi(m - 5 \log r) \tag{f}$$

For D_r the best available analytical form, in my opinion, is that given by Schwarzschild:[1]

$$\log D_r = a_0 + a_1(5 \log r) - a_2(5 \log r)^2 \tag{g}$$

where

$$a_0 = +0.488, \qquad a_1 = +0.097, \qquad a_2 = +0.0088 \tag{h}$$

For the luminosity-curve I used the form following at once from that given in *Astronomical Journal*, No. 566.

$$\phi(M) = \frac{h}{\sqrt{\pi}} e^{-h^2(M-K)^2} \tag{i}$$

[1] *Astronomische Nachrichten*, **190**, 361, 1912.

where

$$h = 0.248, \qquad K = 9.5 \qquad (j)$$

We thus get

$$\log \Delta_{m,r} = a_0 + a_1(5 \log r) - a_2(5 \log r)^2 - h^2 \text{ mod. } (m - K - 5 \log r)^2 \qquad (k)$$

in which the first term, being a constant, may be disregarded. Meanwhile Schwarzschild's form (g) has the defect that it gives D zero for $r = 0$. It cannot, therefore, be accepted for very small values of r. In ordinary cases this defect can scarcely be of any importance; but for the application to the large proper motion stars with which we are more particularly concerned, for all of which r must be small, its use would certainly be objectionable. For these objects I took D_r = constant, which is equivalent to neglecting a_1 and a_2 in (k). Table XXXIV of the text, therefore, was computed for the arguments $v = 0''.25$ and higher by substituting in (d):

$$\log \Delta_{m,r} = -h^2 \text{ mod. } (m - K - 5 \log r)^2 \qquad (l)$$

and then integrating numerically. For $v = 0''.00$ (l) was replaced by (k). For this particular value of v the expressions in (d) become rigorously integrable. We get:

$$\log \overline{\pi}(v = 0''.00) = -1.816 - 0.150m \qquad (m)$$

by the aid of which the first column of Table XXXIV was calculated.

NOTATION USED IN THE LISTS AND IN THE PAPER

The letters used exclusively in Section 4 have there been explained.

- a = absorption of light in magnitudes per unit of distance ($\pi = 0''.1$)
- b = galactic latitude
- h = $0.4769/r$ = modulus of precision in the frequency-curve (45)
- h' = modulus of precision in the frequency-curve (28)
- K = constant correction of observed radial velocities; also median in frequency-curve (45)
- l = galactic longitude
- λ = distance on the sphere: star—vertex
- μ = total proper motion
- μ_α = proper motion in right ascension (arc of great circle)

μ_δ = proper motion in declination

$\left.\begin{array}{l}\delta\mu_\alpha\\ \delta\mu_\delta\end{array}\right\}$ = systematic corrections to Boss's μ_α and μ_δ

m = apparent magnitude

m_0 = limiting apparent magnitude

M = absolute magnitude (= app. mag. star would show at $r=1$)

n = number of stars

p = position angle of proper motion. Also: spectrum peculiar

Δp = $p_o - p_c$

π = parallax

π_1 = parallax computed by formula (38) and elements (27)

π_2 = parallax computed by formula (38) and elements (57), (58), (59)

r = probable error; also distance: $r = 0\rlap{.}''1/\pi$

r_p = $57\rlap{.}^\circ3\, r/\mu$ = probable observation error of position angle

r_π = total probable error of π computed by (38) including observation error in μ

r'_π = total probable error of π computed by (38) excluding observation error in μ

r_ρ = probable error of a radial velocity resting on more than one observation, excluding peculiar motion

r'_ρ = total probable error of a radial velocity resting on more than one observation including both observation error and peculiar motion

$\left.\begin{array}{l}r_u\\ r_v\end{array}\right\}$ = probable amount of the quantities u and v

ρ = radial velocity

$\left.\begin{array}{l}u\\ v\end{array}\right\}$ = components of the peculiar linear motion at right angles to the line of sight, the one toward the antivertex, the other at right angles thereto (see Fig. 3, Section 9)

υ = component of the angular proper motion μ along the great circle toward the antivertex

V = stream-velocity in kilometers per second

C = (in column of spectra) spectrum composite

c = (in column of spectra) spectrum lines sharp

LIST 1

GALACTIC LATITUDE +; $100\mu \geqq 1''.7$

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal. Long.	Gal. Lat.	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0".0001	$\frac{r_\pi}{\pi}$	M	π_2 Unit 0".0001
2600	B3	4.96	9h.6	14°	217°	20°	3".1	238°	+19.0	129°	+ 11°	7°	+ 3.2	......	99	0.20	−0.1	97
2190	B	6.10	8.2	32	218	2	2.1	239	......	155	0	17	......	G	129	42	+1.6	127
2257	B	5.82	8.4	35	223	3	6.5	278	......	154	+ 33	16	......	G	322	44	+3.3	337
2605	B2	4.74	9.6	23	224	23	1.7	295	......	134	(+ 62)	34	......	Excl.	......	......	......	......
2428	B5	5.54	9.0	42	232	3	4.6	267	+32.0*	150	+ 10	10	+11.9	*F*	232	31	+2.3	236
2483	B3	5.15	9.2	43	234	5	3.4	268	......	148	+ 9	11	......	*F*	165	31	+1.2	173
2472	B	5.92	9.1	46	236	2	2.9	258	......	149	− 6	18	......	*F*	137	39	+1.6	147
2790	B9	5.65	10.4	30	238	24	4.5	257	+25.0†	129	+ 18	6	+ 9.2	*F*	142	20	+1.4	146
2640	B8	5.26	9.8	45	240	7	3.4	249	......	142	− 7	13	......	*C*	141	30	+1.0	143
2577	B8	5.16	9.5	51	243	1	3.1	231	......	......	......	16	......	Vela	......	......	......	......
2659	B	6.00	9.8	51	245	3	1.7	216	......	......	......	28	......	Vela	......	......	......	......
2669	B	6.47	9.9	51	245	3	3.2	196	......	......	......	15	......	Vela	......	......	......	......
2674	B5	3.70	9.9	54	247	1	2.3	255	+14.4	143	− 12	11	− 4.5	*C*	99	28	−1.3	94
2755	B5p	4.65	10.3	56	251	1	1.8	270	+ 9.0	139	+ 6	17	− 9.3	*C*	70	36	−1.1	70
2843	B8	6.62	10.6	55	253	3	1.7	211	......	......	......	25	......	Vela	......	......	......	......
2860	B3p	5.44	10.6	59	255	0	2.0	186	0.0*	......	......	22	......	Vela	......	......	......	......
3073	B8	4.68	11.6	34	255	27	3.0	272	......	119	+ 35	10	......	*C*	74	26	−0.9	76
2905	B3	6.64	10.8	58	256	1	2.2	262	......	136	0	20	......	*C*	82	40	+1.2	82
2992	B5	4.26	11.3	54	257	6	3.9	243	......	131	− 9	6	......	*C*	134	20	−0.1	130
3115	B9	4.40	11.8	33	258	28	5.6	264	−20.0*	117	+ 28	4	−32.6	*C*	142	16	+0.2	144
3048	B8	4.82	11.5	54	259	7	6.5	267	+12.0*	128	+ 17	6	− 3.6	*C*	206	19	+1.3	206
3091	B8	5.44	11.7	45	259	16	6.8	270	......	123	+ 29	6	......	*C*	181	20	+1.7	184
3057	B3	5.84	11.5	60	262	2	2.8	293	......	131	+ 37	21	......	*C*	76	49	+0.2	77
3168	B	5.62	12.1	41	263	22	2.9	234	......	118	− 3	17	......	*C*	85	34	+0.3	80
3162	B5	4.81	12.1	50	264	12	3.0	260	+16.4	122	+ 19	12	+ 2.4	*C*	88	27	−0.5	86
3165	B3	2.88	12.1	50	264	12	4.3	246	......	122	+ 5	4	......	*C*	130	17	−1.6	130
3127	B5	5.70	11.9	62	264	0	1.9	201	......	......	......	25	......	Vela	......	......	......	......
3227	B9	5.42	12.3	35	264	27	4.4	235	......	112	+ 2	10	......	*C*	122	22	+0.8	119
3176	B3	4.20	12.1	52	265	10	4.9	243	+28.0	123	0	6	+13.7	*C*	150	19	+0.1	150
3232	B9	5.77	12.3	35	265	27	4.3	252	......	112	+ 19	9	......	*C*	114	21	+1.2	113
3187	B3	3.08	12.2	58	266	4	4.8	246	+25.0	125	− 1	4	+10.2	*C*	151	18	−1.0	148
3239	B3	5.04	12.3	51	266	12	7.8	242	......	120	+ 2	6	......	*C*	231	18	+1.8	233
3249	B9	5.60	12.4	38	266	24	3.1	238	......	113	+ 4	8	......	*C*	87	20	+0.3	82
3236	B5	5.14	12.3	63	267	0	5.4	222	......	127	− 26	6	......	*C*	158	20	+1.1	148
3237	B1	1.58	12.3	62	267	0	4.7	228	+ 7.0	127	− 20	4	− 8.3	*C*	142	18	−2.7	133
3238	B1	2.09	12.3	62	267	0	4.9	251	......	127	+ 3	4	......	*C*	158	18	−1.9	151
3245	B3	4.16	12.4	50	267	13	4.7	233	......	118	− 5	6	......	*C*	138	18	−0.1	135
3300	B8	4.79	12.6	39	269	24	6.3	227	+13.0	112	− 6	5	+ 1.8	*C*	172	17	+1.0	170
3314	B8	6.25	12.6	56	269	7	4.8	236	......	121	− 4	7	......	*C*	144	0.20	+2.0	138

3305	B8	5.02	12h6	59°	270°	4°	2°5	241°	+19.5	122°	− 1°	15°	+ 5.5	C	76	0.30	−0.6	73
3312	B9	6.23	12.6	55	270	8	5.3	231	....	120	− 9	7	....	C	154	20	+2.2	150
3328	B1	1.50	12.7	59	270	4	5.6	240	+13.0	122	0	3	− 1.0	A II‡	170	16	−2.2	165
3324	B3	4.86	12.7	56	270	7	5.5	224	+18.2	120	− 15	7	+ 4.8	A II	157	18	+0.8	153
3345	B8	6.14	12.8	39	271	24	5.3	224	....	110	− 7	7	....	A II	142	18	+1.8	138
3357	B3	4.84	12.8	59	271	4	5.1	219	....	121	− 20	9	....	A II	144	21	+0.6	139
3358	B3	4.26	12.8	57	271	6	3.6	248	+13.2	120	+ 9	9	− 0.2	A I	107	21	−1.1	105
3359	B3	5.46	12.8	57	271	6	2.7	231	+ 7.0*	120	− 8	16	− 6.4		83	32	+0.1	75
3369	B8	5.29	12.8	51	271	12	5.2	225	....	117	− 10	9	....	A II	148	21	+1.1	143
3366	Oe5	5.58	12.8	56	271	7	4.5	235	....	119	− 3	9	....	A II	133	21	+1.2	128
3390	B3	4.96	13.0	48	273	15	5.9	221	....	113	− 12	6	....	A II	162	18	+1.0	159
3393	B3	4.40	13.0	49	273	13	4.2	235	....	114	+ 1	6	....	A II	119	18	−0.2	115
3419	B8	4.76	13.1	59	273	4	6.1	236	....	118	− 1	7	....	A II	179	19	+1.0	175
3457	B5	4.62	13.3	60	274	2	3.9	241	....	118	+ 6	11	....	A I	113	24	−0.1	112
3456	B3	6.51	13.3	60	274	2	2.9	217	....	118	− 18	15	....	A I	82	30	+1.1	77
3451	B3	5.70	13.3	52	275	10	5.7	213	....	114	− 19	6	....	A II	153	18	+1.6	145
3458	B	6.10	13.3	52	275	10	2.9	274	....	114	+ 42	20	....	....	60	50	0.0	62
3521	B1	2.56	13.6	53	278	9	4.1	229	+ 6.0	112	0	5	− 5.2	A II	114	17	−2.2	109
3528	B9	5.40	13.6	54	278	8	6.8	218	....	113	− 12	6	....	A II	188	18	+1.8	182
3535	B	6.30	13.6	56	278	6	2.4	210	....	115	− 20	16	....	A I	65	32	+0.3	56
3576	B8	5.42	13.8	52	280	9	6.7	228	....	110	+ 1	5	....	A II	184	16	+1.7	180
3592	B8	5.84	13.8	52	280	9	5.8	236	....	110	+ 9	8	....	A II	156	20	+1.8	152
3615	B1	0.86	13.9	60	280	1	4.1	219	+12.0*	114	− 9	3	+ 0.3	A II	116	15	−3.8	111
3575	B3	5.87	13.8	46	281	15	4.4	208	....	107	− 18	10	....	A II	113	23	+1.1	109
3565	B2	3.32	13.7	42	282	19	2.8	230	+12.3	105	+ 4	9	+ 3.3	A I	74	21	−2.3	70
3593	B2p	3.06	13.8	47	282	14	8.2	230	....	107	+ 5	2	....	A II	220	14	−0.2	217
3643	B3	5.20	14.1	57	282	4	3.6	247	....	112	+ 21	11	....	A I	91	26	0.0	92
3564	B2	3.53	13.7	41	283	20	4.2	234	+ 6.0†	104	+ 8	6	− 2.7	A II	112	17	−1.2	107
3602	B3	4.05	13.9	42	283	19	3.4	233	+ 5.3	103	+ 9	8	− 3.1	A I	90	19	−1.2	85
3670	B5	4.41	14.2	56	283	4	3.2	229	+ 3.5	111	+ 4	12	− 7.4	A I	89	26	−0.8	83
3603	B3	4.17	13.9	44	284	17	4.6	226	+ 4.0*	104	+ 2	5	− 4.7	A II	122	16	−0.4	119
3577 3578	B5	4.72	13.8	32	286	29	6.0	230	+14.1	97	+ 7	4	+ 7.6	A II	154	15	+0.7	150
3586	B5	4.76	13.8	31	286	30	2.8	221	+ 8.0†	96	− 2	9	+ 1.8	A I	73	21	−0.9	68
3621	B3	4.54	14.0	41	286	19	3.8	208	+12.4	102	− 15	6	+ 4.3	A I	97	18	−0.5	85
3732	B5	4.14	14.5	49	287	10	5.1	223	....	103	+ 3	7	....	A II	133	18	−0.2	132
3699	B3	4.65	14.3	45	287	14	3.0	197	−16.6	102	− 24	8	−24.7	....	71	21	−1.1	67
3716	B2	4.60	14.4	50	287	9	5.4	256	− 0.6	105	+ 35	5	− 9.6	....	117	16	−0.1	116
3713	B9	5.49	14.4	45	288	14	6.8	219	....	101	− 1	5	....	A II	179	16	+1.8	176
3688	B5	4.55	14.3	39	289	20	4.9	215	+ 9.2	97	− 5	5	+ 2.7	A II	128	16	+0.1	124
3745	B2	2.89	14.6	47	289	11	3.5	217	+ 8.0†	102	− 1	6	− 0.1	A I	92	19	−2.3	88
3724	B3C	2.65	14.5	42	290	16	4.9	224	0.0	98	+ 6	4	− 6.9	A II	128	0.15	−1.8	124

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

‡ Also used in C.

LIST 1.—*Continued*

Boss No.	Sp.	Harv. Mag.	1900		Gal. Long.	Gal. Lat. +	100 μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0″.0001	r_π/π	M	π_2 Unit 0″.0001
			α	$\delta-$														
3808	B8	5.78	14ʰ.8	48°	291°	10°	3″.7	223°	......	99°	+ 7°	12°	......	A I	96	0.25	+0.7	93
3747	B3	4.09	14.6	37	293	20	4.4	204	......	93	− 13	6	......	A II	111	17	−0.7	105
3783	B5	4.49	14.7	43	293	14	5.0	225	+ 7.2	97	+ 9	6	+ 0.7	A II	128	17	0.0	126
3838	B5	4.72	15.0	47	293	10	4.0	215	......	99	0	6	......	A II	104	16	−0.2	100
3888	B8	4.36	15.2	48	294	8	5.9	212	......	98	− 1	5	......	A II	154	16	+0.3	148
3815	B2p	2.81	14.9	43	294	13	6.9	223	0.0†	96	+ 7	3	− 6.2	A II	177	14	−1.0	173
3862 } 3863 }	B9, A	4.14	15.1	48	294	8	12.1	237	......	98	+ 23	2	......	A II	289	14	+1.4	288
3852	B3	4.39	15.0	45	294	11	3.1	230	......	97	+ 16	11	......	A I	78	24	−1.1	73
3707	B8	5.00	14.4	29	295	29	4.2	217	......	89	− 2	9	......	A II	108	16	+0.2	105
3818	B3	3.35	14.9	42	295	14	3.6	208	+10.0†	96	− 8	6	+ 3.8	A I	94	17	−1.8	87
3796	B8	5.11	14.8	37	295	19	4.3	242	......	84	+ 26	9	......	A II	102	22	+0.2	100
3965	B9	5.48	15.5	52	295	2	5.4	218	......	100	+ 8	9	......	A II	140	21	+1.2	135
4011	B9	5.96	15.7	53	295	0	5.4	223	......	101	+ 14	8	......	A II	138	20	+1.7	133
3865	B3	4.92	15.1	44	296	11	5.1	230	......	96	+ 16	8	......	A II	128	21	+0.5	123
3905	B3	3.74	15.3	44	297	10	2.8	215	+14.0†	94	+ 3	9	+ 8.4	A I	72	21	−2.0	68
3892	B8	5.78	15.2	40	298	14	4.1	218	+21.0*	92	+ 6	9	+16.1	A II	106	21	+0.9	102
3954	B3	4.84	15.5	45	298	8	3.9	189	+ 5.0†	94	− 21	12	− 0.6	A I	93	27	−0.4	86
3896	B2	3.43	15.2	40	299	14	3.3	192	......	92	− 20	8	......	A I	80	21	−2.0	73
3832	B3	5.45	15.0	32	300	22	6.4	217	......	88	0	7	......	A II	165	18	+1.5	163
3776	B9	5.80	14.7	26	301	29	2.6	216	......	85	− 1	11	......	A I	67	23	−0.1	63
3950	B3	2.95	15.5	41	301	11	3.9	204	......	91	− 6	5	......	A I	100	15	−2.0	94
3910	B3	4.69	15.3	36	302	17	3.8	215	......	88	+ 4	6	......	A I	98	17	−0.4	94
4128	B4	5.36	16.1	47	302	2	6.6	207	......	94	+ 3	6	......	A II	170	17	+1.5	165
3929	B5	5.52	15.4	36	302	16	4.1	183	......	88	− 29	9	......	......	93	22	+0.4	87
4190	B1p	5.46	16.4	46	304	1	1.9	238	−21.0*	92	+ 37	23	−25.9	......	39	53	−1.6	37
3991	B5	4.82	15.6	34	306	16	4.9	221	+12.0*	85	+ 12	7	+ 9.3	A III§	124	18	+0.3	121
4064	B3	3.61	15.9	38	307	11	4.4	207	......	86	+ 1	5	......	A III	113	16	−1.1	108
4076	B5	4.97	15.9	38	307	10	4.4	217	+ 5.0*	86	+ 12	8	+ 2.0	A III	111	20	+0.2	108
4006	B8	5.61	15.7	34	307	15	2.7	214	......	84	+ 7	16	......	A III	70	31	−0.2	63
4018	B9	4.11	15.7	33	308	16	2.9	198	......	82	− 9	8	......	A III	76	19	−1.5	69
4224	B	6.14	16.5	43	308	2	4.8	220	......	88	+ 20	7	......	A III	116	20	+1.5	113
4091	B3	4.33	16.0	36	308	11	4.1	211	......	85	+ 6	5	......	A III	106	16	−0.5	100
3973	B3	3.80	15.5	29	309	20	4.1	216	......	81	+ 7	5	......	A III	107	16	−1.0	104
4287	B1p	4.88	16.8	42	311	1	2.0	163	−27.0	......	(− 34)	18	(−30.0)	Excl.	......	......	......	......
4248	B3	5.68	16.6	41	311	2	4.0	228	......	86	+ 29	9	......	A III	82	24	+0.2	87
4052	B3	4.02	15.8	29	312	18	3.3	203	+ 3.2	79	− 4	5	+ 2.4	A III	87	16	−1.3	80
4019	B3	4.77	15.7	25	314	21	4.2	213	......	75	+ 6	5	......	A III	111	16	0.0	107
4200	B3	4.33	16.4	34	314	9	2.6	200	......	80	− 1	10	......	A III	68	22	−1.5	61
4277	B3p	3.09	16.7	38	314	3	3.1	193	......	83	− 5	6	......	A III	81	17	−2.4	74
4281	B2	3.64	16.8	38	314	3	3.4	214	+ 1.7	83	+ 16	7	− 0.4	A III	86	0.18	−1.7	82

4034	B3	4.66	15h.8	25°	315°	21°	3".4	200°	....	75°	− 7°	5°	....	*A* III	90	0.16	−0.6	85
4062	B2p	3.00	15.9	26	315	19	3.9	203	....	75	− 3	4	....	*A* III	104	15	−1.9	99
3944	B3	5.59	15.5	16	316	30	2.8	207	Bin.	70	− 5	6	....	*A* III	77	17	0.0	70
4058	B8	5.41	15.9	24	316	20	5.2	237	....	75	+ 30	10	....	*A* III	120	24	+0.8	121
4115	B3	4.70	16.1	28	317	16	4.7	216	....	76	+ 11	4	....	*A* III	122	15	+0.1	120
4066	Bp	2.54	15.9	22	318	22	4.1	198	....	72	− 9	3	....	*A* III	111	14	−2.2	106
4033	B3	5.06	15.8	20	319	25	3.7	202	Bin.	71	− 6	4	....	*A* III	100	15	+0.1	95
4158	B1	3.08	16.2	25	319	17	3.4	200	....	73	− 3	4	....	*A* III	91	15	−2.1	86
4218	B	2.91	16.5	28	319	12	3.9	197	+ 1.5	74	− 3	3	+ 2.2	*A* III	105	14	−2.0	99
4198	B3	4.87	16.4	25	320	15	2.8	188	....	72	− 14	8	....	*A* III	73	20	−0.8	67
4321	B9	5.06	16.9	32	320	5	6.3	190	....	76	− 6	5	....	*A* III	165	16	+1.1	161
4178	B5	5.22	16.3	23	321	17	2.4	208	....	71	+ 5	6	....	*A* III	65	18	−0.7	59
4086	B1	2.90	16.0	20	321	23	3.1	201}	− 9.5‡	70	+ 6	3	− 7.5	*A* III	85	14	−2.4	85
4087	B1	5.06	16.0	20	321	23	3.4	223}										
4093	B2	4.13	16.0	20	321	22	3.4	192	−11.0	70	− 14	5	− 9.0	*A* III	90	16	−1.1	85
4117	B3	4.29	16.1	19	323	22	3.4	200	....	69	− 5	4	....	*A* III	94	16	−0.8	88
4059	B3	4.68	15.9	14	324	28	3.4	206	....	66	− 2	5	....	*A* III	96	17	−0.4	90
4183	B3	4.85	16.3	18	325	20	3.1	188	Bin.	66	− 15	6	....	*A* III	85	18	−0.5	79
4399	B3	3.37	17.3	25	328	6	3.1	182	− 0.9	68	− 10	4	+ 1.7	*A* III	83	16	−2.0	77
4495	B3	6.16	17.7	27	329	0	2.5	217	....	68	+ 30	12	....	*A* III	61	29	+0.1	57
4225	B	2.70	16.5	10	334	23	2.1	35	....	....	(−169)	6	....	Excl.	....	....	....	....
4465	B8	4.65	17.5	8	344	11	2.5	189	....	52	− 3	7	....	*A* III	82	21	−0.8	75
4573	B8	5.79	18.0	8	348	5	2.6	126	− 5.3	50	− 58	16	+ 2.2	*A* III	47	55	−0.9	46
4302	B8	4.29	16.8	10‖	357	29	7.3	231	....	38	+ 24	2	....	*A* III	278	20	+1.4	283
4545	B3	4.81	17.9	4‖	359	12	1.9	205	−11.0	38	+ 18	10	− 0.9	*A* III	75	0.28	−0.8	67

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

‡ Velocity center of mass from orbit.

§ Also used in *A* III.

‖ Declination north.

LIST 2

GALACTIC LATITUDE −; $100\mu \geqq 1''.7$

Boss No.	Sp.	Harv. Mag.	1900 α	1900 δ−	Gal. Long.	Gal. Lat.	100 μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0''.0001	r_π/π	M	π_2 Unit 0''.0001
1910	B3	5.11	7h2	37°	217°	10°	1''.7	220°	+18.0	167°	− 27°	21°	− 4.1	G	177	......	+1.4	165
2096	B8	5.41	7.8	36	219	3	3.7	189	......	......	......	12	......	Vela	......	......	......	......
1702	B8	3.18	6.6	43	219	19	2.0	177	+26.0*	177	−103	9	+ 3.4	G	3° from antivertex			
2017	B5	4.91	7.6	38	220	7	2.3	255	+24.1	164	+ 3	15	+ 2.2	G	211	0.52	+1.5	222
2018	A †	5.74	7.6	38	220	7	2.5	265	......	164	+ 13	20	......	G	221	56	+2.5	243
2019	B5	5.78	7.6	38	220	7	2.4	263	......	164	+ 11	15	......	G	221	52	+2.5	243
2026	B8	5.48	7.6	38	220	6	2.4	280	......	164	+ 28	15	......	G	193	54	+1.9	222
2038	B5	6.44	7.7	38	220	6	2.8	255	......	164	+ 3	13	......	G	257	50	+3.4	257
2048	B8	5.86	7.7	38	220	6	2.6	270	......	164	+ 18	15	......	G	230	52	+2.6	238
2060	B3	5.11	7.7	38	220	5	1.7	126	......	164	(−126)	21	......	Excl.	......	......	......	......
2141	Od	2.27	8.0	40	224	4	3.5	282	......	160	+ 25	5	......	G	243	38	−0.8	266
2109	B5	5.42	7.9	43	225	6	1.9	32	......	162	(+127)	34	......	Excl.	......	......	......	......
2062	B2	5.26	7.8	46	227	9	2.5	253	......	163	− 23	21	......	F	204	60	+1.8	196
2165	B3	5.16	8.1	44	228	5	2.2	153	......	159	(−112)	27	......	Excl.	......	......	......	......
2095	B1	4.32	7.8	48	230	10	1.8	186	......	......	......	17	......	Vela	......	......	......	......
1558	B1	4.84	6.1	55	230	27	2.4	250	......	166	(−118)	12	......	Excl.	......	......	......	......
1925	B	5.50	7.3	52	231	16	3.6	264	......	165	− 46	21	......	F	248	73	+2.4	238
2119	B1p	4.50	7.9	49	231	9	3.0	258	......	162	− 24	14	......	F	225	49	+1.2	210
2166	B3	4.79	8.1	47	231	7	1.7	205	......	......	......	16	......	Vela	......	......	......	......
2171	B3	5.40	8.1	48	231	7	2.6	198	......	......	......	21	......	Vela	......	......	......	......
2191	B3	5.28	8.2	47	231	6	3.8	208	......	......	......	20	......	Vela	......	......	......	......
2127	B3	6.43	7.9	50	232	10	1.7	253	......	162	− 32	27	......	F	125	68	+1.9	113
2128	B3	6.65	7.9	50	232	10	2.1	305	......	162	+ 20	25	......	F	167	62	+2.8	177
2226	B2	4.90	8.3	48	232	5	1.7	263	+19.0	157	− 10	25	− 2.1	F	113	54	+0.1	114
2270	B	6.11	8.4	46	232	3	4.6	251	......	157	− 18	8	......	F	292	35	+3.4	277
2360	B5c	5.54	8.7	46	233	1	3.4	195	......	......	......	13	......	Vela	......	......	......	......
2111	B3	3.60	7.9	53	234	12	3.8	297	+19.6	160	+ 3	5	− 1.9	F	288	38	+0.9	298
2382	B	4.89	8.8	46	234	1	2.3	211	......	......	......	21	......	Vela	......	......	......	......
2250	B2	5.23	8.4	51	235	7	4.7	258	......	157	− 22	13	......	F	292	40	+2.5	277
2337	B3	5.48	8.6	48	235	3	4.8	257	......	155	− 13	12	......	F	288	38	+2.8	275
2041	B	6.44	7.7	58	238	16	2.5	258	......	158	− 54	17	......	E	104	45	+1.4	90
2319	B5	5.60	8.6	53	238	6	3.7	301	......	155	+ 21	13	......	E	209	39	+2.1	208
2322	B5	5.38	8.6	53	238	6	4.4	263	......	155	− 17	9	......	E ‡	258	35	+2.6	243
2325	B3	3.68	8.6	53	238	6	2.0	307	+18.0*	155	+ 27	14	− 2.9	E	111	40	−1.1	116
2340	B9	5.68	8.6	53	238	6	3.8	295	......	154	+ 15	12	......	E	216	36	+2.3	220
2341	B5	5.04	8.7	53	238	6	2.6	274	+19.4	153	− 6	17	− 1.2	E	149	42	+0.9	145
2433	B9	5.42	9.0	52	240	3	2.2	306	......	151	+ 32	19	......	E	102	47	+0.4	105
2225	B5	6.07	8.3	58	241	11	3.6	267	......	154	− 29	11	......	E	181	36	+2.4	179
2298	B3	5.40	8.5	58	241	10	4.9	278	......	153	− 12	9	......	E	274	0.33	+2.5	259

2136	B3	4.96	8h0	63°	243°	16°	2°0	319°	....	154°	+ 5°	21°	....	*E*	117	0.46	+0.3	123
2331	B2	4.42	8.6	59	243	10	2.5	256	....	152^{s}	− 35	13	....	*E*	110	40	−0.4	108
1469	B5	5.15	5.8	67	244	30	2.7	295	+18.0	155	(− 80)	12	− 2.9	Excl.	......	......	......	......
2526	B3	2.63	9.3	55	244	3	2.1	256	+21.9§	147^{s}	− 19	11	+ 2.2	*E*	96	31	−2.5	89
2406	B8	3.98	8.9	60	245	9	6.2	336	+24.5	150	+ 48	7	+ 4.4	*E*	211	31	+0.6	218
1566	B9	5.21	6.2	69	246	28	6.7	298	+25.0‖	153	(− 65)	5	+ 4.4	Excl.	......	......	......	......
2473	B3	3.56	9.1	59	246	7	4.1	263	+23.3§	149	− 20	7	+ 3.3	*E*	193	27	0.0	188
2581	B5	4.20	9.5	59	247	5	1.8	308	+21.7	146	+ 30	15	+ 2.2	*E*	74	38	−1.5	76
2477	B3	4.18	9.2	62	248	9	4.7	276	+16.0	145	− 4	7	− 3.3	*E*	213	25	+0.8	215
1795	B8	5.52	6.9	71	249	25	2.9	344	....	151	− 4	14	....	*E*	156	36	+1.5	158
2179	B5	4.46	8.1	68	249	18	2.8	321	....	150	+ 3	12	....	*E*	144	33	+0.3	144
2604	B9	4.67	9.6	61	250	6	3.9	299	+26.0*	146	+ 19	9	+ 6.5	*E*	170	29	+0.9	169
2228	B9	5.44	8.3	71	252	18	4.6	319	....	147	+ 2	8	....	*E*	220	27	+2.1	224
2811	B5	3.58	10.5	61	255	2	2.6	286	+24.0	138	+ 17	11	+ 6.1	*E*	96	28	−1.4	100
2859	B3	5.20	10.6	64	257	4	1.7	294	....	137^{s}	+ 24	20	....	*E*	57	45	−1.0	61
2862	B	3.03	10.6	64	257	4	2.4	293	+16.0	137^{s}	+ 23	8	− 1.8	*E*	83	25	−2.4	87
2876	B5	5.54	10.7	64	257	4	3.8	209	....	......	....	13	....	Vela	......	......	......	......
2733	B8	3.56	10.2	70	258	11	2.8	266	....	140	− 16	9	....	*D*	109	26	−1.2	103
2969	B9	5.52	11.2	64	260	3	2.8	257	....	133^{s}	− 6	15	....	*D*	99	33	+0.5	95
2606	B	5.24	9.6	80	262	20	4.4	279	....	137	− 23	6	....	*D*	155	22	+1.1	148
3054	B9	3.34	11.5	62	262	0	4.8	242	+11.0‖	132	− 15	5	− 5.5	*D*	164	19	−0.7	157
2103	B5	4.52	11.7	63	264	1	2.0	225	....	130	− 31	21	....	*D*	57	47	−1.7	50
3889	B3	4.62	10.7	80	265	19	4.8	265	+22.0	134^{s}	− 19	5	+ 4.9	*D*	165	21	+0.7	162
3111	B8	5.10	11.8	65	265	3	3.9	247	....	130	− 9	10	....	*D*	131	24	+0.7	125
3200	B3	4.26	12.2	63	266	1	5.0	247	+19.9	127	− 2	8	+ 4.6	*D*	161	21	+0.3	157
3134	B9	5.05	11.9	78	268	16	4.4	247	....	132	− 19	6	....	*D*	146	20	+0.8	140
3199	B5	4.38	12.2	79	268	16	4.9	284	....	131	+ 20	3	....	*D*	158	18	+0.4	160
3289	B3	2.94	12.5	69	269	6	4.2	240	+13.5	130	− 10	7	− 2.6	*D*	137	21	−1.4	125
3269	B5	4.04	12.4	72	269	9	4.6	253	....	129	0	7	....	*D*	152	20	+0.1	146
3320	B3	3.26	12.7	68	270	5	4.1	225	....	126	− 22	7	....	*D*	121	21	−1.3	115
3429	B8	4.95	13.1	67	273	.4	3.2	234	....	123	− 7	11	....	*D*	98	25	0.0	93
3646	B3	5.88	14.2	66	279	5	1.7	239	....	118	+ 11	30	....	*D*	50	54	−0.6	44
3925	B5p	5.65	15.3	73	281	14	2.3	182	....	120	− 36	17	....	*D*	56	39	−0.7	47
3956	B8	5.76	15.5	73	281	14	2.4	222	....	119	+ 7	16	....	*D*	71	29	0.0	64
3886	Oe5	5.50	15.2	60	287	3	2.5	233	....	108	+ 18	19	....	*D*	65	38	−0.5	60
3874	Oe5	5.24	15.2	61	287	3	2.2	216	....	109	0	17	....	*D*	60	33	−0.9	55
3872	B1	5.95	15.2	60	287	3	2.2	256	....	109	+ 40	17	....	*D*	46	41	−0.8	44
3901	B5C	4.54	15.2	59	289	2	4.3	193	....	107	− 22	9	....	*B*	108	21	−0.3	100
4268	B9	6.30	16.7	65	292	13	2.3	195	....	108	− 3	17	....	....	62	34	+0.2	54
4013	B5	5.84	15.7	55	294	1	3.7	163	....	102	− 46	13	....	*B*	68	35	0.0	61
4094	B9	5.79	16.0	58	294	5	6.5	207	+ 6.0‖	104	+ 1	7	− 2.7	*B*	173	0.18	+2.0	166
4392	B3	5.88	17.2	63	296	15	2.4	118	....	107	(− 74)	23	....	Excl.	......	......	......	......

* Spect. Binary, estimated velocity of center of mass.

† Inserted because of community of motion with Nos. 2017, 2019, 2026, 2038, 2048.

‡ Also used in *F*.

§ Velocity center of mass from orbit.

‖ 1 observation.

LIST 2.—*Continued*

Boss No.	Sp.	Harv. Mag.	1900		Gal. Long	Gal. Lat. −	100 μ	p	ρ	λ	$\overset{p}{\text{O}-\text{C}}$	r_p	$\overset{\rho}{\text{O}-\text{C}}$	Group	π_1 Unit 0".0001	$\frac{r_\pi}{\pi}$	M	π_2 Unit 0".0001
			α	$\delta-$														
4253	B	5.76	16h6	58°	297°	9°	5".2	209°	....	102°	+ 10°	10°	....	*B*	134	0.21	+1.4	128
4249	B3	5.94	16.6	58	297	9	3.0	180	0.0*	102	− 19	16	− 8.1	*B*	76	32	+0.3	69
4426	B8	3.79	17.4	61	298	15	10.2	213	....	104	+ 23	3	....	*B*	250	14	+0.7	243
4324	B3	5.88	16.9	58	299	10	3.2	206	....	102	+ 10	13	....	*B*	82	28	+0.4	74
4762	B2	4.42	18.7	62	300	24	1.9	235	....	104	+ 59	14	....	....	27	48	−3.4	24
4666	B8	4.81	18.4	62	300	22	3.4	192	....	104	+ 13	9	....	*B*	88	21	−0.4	80
4156	B5	5.49	16.2	49	302	0	3.3	200	+ 1.0*	95	− 2	15	− 4.9	*B*	85	31	+0.1	79
4296	B8	6.57	16.8	50	304	5	3.1	216	....	94	+ 19	15	....	*B*	77	30	+1.0	73
4611	B5	5.54	18.2	56	305	18	4.7	236	....	98	+ 53	9	....	*B*	73	29	−0.2	71
4431	B3	2.97	17.4	50	308	9	9.0	201	+ 2.0†	93	+ 11	2	− 3.3	*B*	227	14	−0.3	223
4398	B3	5.50	17.3	47	310	7	4.5	171	....	91	− 21	11	....	*B*	108	25	+0.7	102
4796	B9	5.03	18.8	53	310	23	2.3	52	− 4.2	95	(−122)	19	−10.1	Excl.	....	....	....	....
4565	B1ac	3.90	18.0	50	311	14	3.0	203	+ 2.8	92	+ 19	8	− 2.1	*B*	75	20	−1.7	68
4404	B8	5.10	17.3	44	313	5	4.0	217	....	87	+ 26	12	....	*B*	93	26	−0.1	90
4677	B8	5.05	18.4	46	315	16	4.0	201	....	88	+ 22	9	....	*B*	95	22	−0.1	89
4657	B3	3.76	18.3	46	315	16	5.3	191	− 0.8	88	+ 11	5	− 4.5	*B*	134	16	−0.6	129
4680	B5	5.33	18.4	46	316	17	1.9	196	....	88	+ 17	17	....	....	46	35	−1.4	39
4429	B3	2.80	17.4	37	319	2	4.2	183	+17.0†	80	− 7	5	+15.9	*B*	111	16	−2.0	105
4439	B2	1.71	17.4	37	319	3	3.6	186	+ 3.0*	80	− 3	5	+ 1.9	*B*	95	15	−3.4	89
4474	B2	2.51	17.6	39	319	5	2.8	201	....	82	+ 13	6	....	....	70	18	−3.3	64
4760	B9	5.64	18.7	44	319	19	2.5	154	....	85	− 22	15	....	....	60	33	−0.5	53
4929	B8	4.24	19.2	45	320	25	1.9	183	− 4.0	87⁵	+ 14	13	− 6.8	....	46	27	−2.5	39
4599	B3	5.52	18.1	41	320	11	5.7	256	....	84	(+ 73)	17	....	Excl.	....	....	....	....
4509	B8	5.97	17.8	35	323	5	1.7	225	....	77	+ 39	24	....	....	35	57	−1.3	33
4690	B9	6.55	18.4	39	323	14	2.8	176	....	81	− 2	12	....	*B*	73	30	+0.8	64
4691	B9	5.95	18.4	39	323	14	3.8	203	....	81	+ 25	11	....	*B*	96	23	+0.8	91
4498	B8	4.83	17.7	32	325	3	2.4	173	−16.0	74	− 14	12	−15.3	....	62	26	−1.3	55
4936	B8	4.11	19.3	41	325	24	12.9	168	....	84	0	2	....	*B*	336	14	+1.7	334
4637	B8	5.39	18.3	37	325	11	2.4	152	−17.0*	78	− 28	15	−17.5	*B*	55	33	−0.9	51
5060	B9	5.39	19.7	40	327	29	3.8	130	....	84	− 34	11	....	*B*	83	27	0.0	80
4792	B5	5.41	18.8	38	327	18	2.9	127	+ 6.0*	80	− 47	12	+ 4.9	....	50	36	−1.1	48
4734	B3	4.82	18.6	36	327	15	5.3	180	+ 1.4	78	+ 3	9	+ 0.9	*B*	139	20	+0.5	136
4913	B5	5.61	19.2	36	330	22	7.4	257	....	79	(+ 88)	16	....	Excl.	....	....	....	....
5108	B3	4.39	19.9	36	332	29	4.7	160	....	81	− 2	10	....	*B*	122	21	−0.2	118
5040	B6	5.56	19.7	32	335	26	2.9	192	....	76	+ 28	11	....	....	66	26	−0.3	63
4739	B8	3.30	18.6	27	335	12	4.8	94	....	68	(− 83)	3	....	Excl.	....	....	....	....
4784	B3	2.14	18.8	26	337	13	6.6	173	− 1.0	68	− 1	1	+ 1.6	*B*	183	35	−1.6	182
4718	B9	5.75	18.5	24	337	9	3.0	195	....	65	+ 17	7	....	*B*	82	18	+0.2	76
4999	B9	4.66	19.5	25	342	22	7.6	109	−17.0†	68	(− 57)	1	−14.4	Excl.	....	....	....	....
4783	B5	5.04	18.8	16	347	9	2.4	250	− 5.7	58	(+ 77)	16	− 0.3	Excl.	....	....	....	....
4816	B5	5.36	18.9	13	349	9	2.5	182	−12.7	56	+ 10	15	− 6.8	....	75	32	−0.2	72
4954	B8	5.68	19.3	15	351	15	2.3	95	−13.0	59	(− 72)	13	− 7.9	Excl.	....	....	....	....
4883	B3	5.37	19.1	8	356	9	1.7	156	−11.5	51	− 13	9	− 4.3	....	53	0.23	−1.0	49

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

LIST 3

HELIUM STARS FOR WHICH $100\mu < 1''.7$

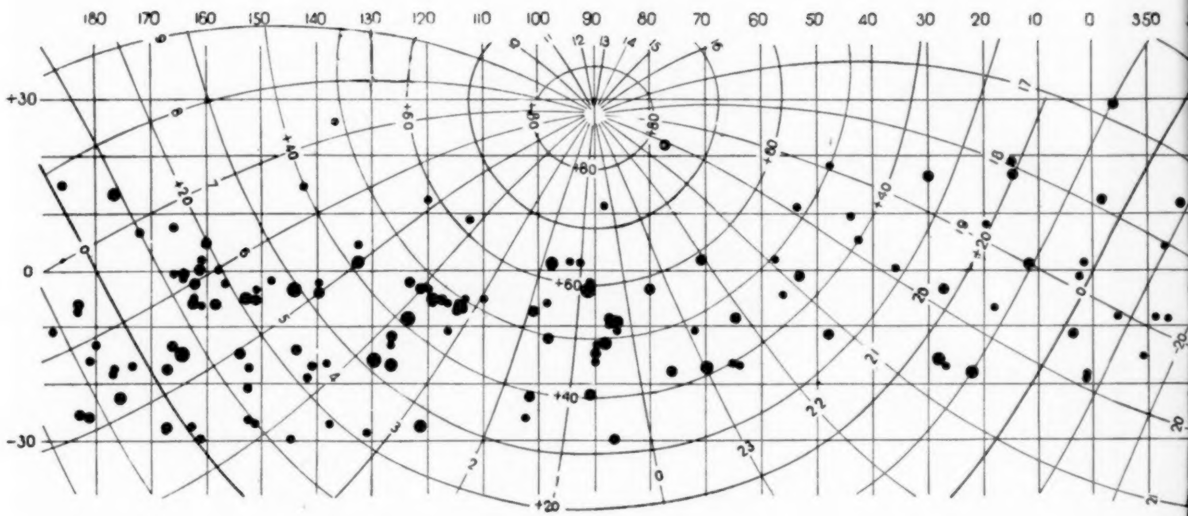
Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal Long.	Gal. Lat.	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0".0001	r_π/π	M	π_2 Unit 0".0001
1907	B3	4.68	7h.2	37°	217°	−10°	1".5	208°		167°	− 39°	22°			148	0.74	+0.5	114
1845	B3	4.85	7.1	40	218	−13	1.3	283	+19.0	170	+ 25	20	− 3.3		182	83	+1.1	217
2188	B3	5.12	8.2	36	221	0	0.9	125		156	−122	46						
2187	B3	4.77	8.2	36	221	0	1.3	180		156	− 67	23			57	66	−1.4	49
2089	B3	4.53	7.8	39	222	− 5	1.6	210		162	− 45	16			92	58	−0.7	86
2342	B2	3.70	8.7	33	223	+ 7	1.3	299	+17.0	150	+ 57	18	− 3.1		52	48	−2.7	60
2217	B3	5.17	8.3	36	223	+ 1	1.4	188		156	− 60	29			70	71	−0.6	54
1719	B	6.47	6.6	48	225	−21	1.4	321		174	− 19	31			6° from antivertex			
2249	B	5.30	8.4	42	227	− 1	1.5	259		156	− 1	24			94	53	+0.2	94
1884	B8	4.88	7.2	48	227	−15	1.6	38		168	(+103)	33		Excl.				
2070	B	4.25	7.8	46	228	− 9	1.4	231	+20.5	163	− 44	23	− 1.3		89	71	−0.9	77
2267	B	5.06	8.4	44	230	− 2	1.2	194		156	− 71	38			50	1.03	−1.4	40
2196	B	6.02	8.2	46	230	− 5	1.1	248		158	− 19	35			70	75	+0.2	70
2094	B3	4.83	7.8	49	230	−10	1.5	293		162	+ 8	36			125	75	+0.3	136
2332	B5	5.23	8.6	45	232	− 1	1.1	221		153	− 44	52			46	1.27	−1.5	36
2266	B5	5.52	8.4	48	233	− 4	1.3	247	0.0	156	− 27	13	−21.0		75	57	−0.1	71
2467	B5	4.96	9.1	44	235	+ 3	1.4	270	+35.0	149	+ 9	27	+15.0		70	54	−0.8	73
2408	B5	4.77	8.9	52	239	− 3	1.1	195		151	− 80	38			38	1.06	−2.3	27
2629	B	5.68	9.7	44	240	+ 8	1.1	270		144	+ 14	34			48	63	−0.9	48
2363	B3	4.63	8.7	56	240	− 7	0.5	259		153	− 26	94						
2575	B3	5.35	9.5	49	241	+ 3	1.5	340		145	+ 75	28			45	77	−1.4	56
2414	B3	5.08	8.9	59	244	− 8	1.2	280		150	− 5	20			62	43	−0.9	66
2702	B3	5.10	10.1	51	246	+ 4	1.0	241		141	− 21	37			37	74	−2.1	33
2457	B3	4.86	9.1	70	254	−15	0.8	310	+26.0*	144	+ 9	62	+ 6.9					
2880	B5	5.10	10.7	64	257	− 4	1.4	210		138	− 40	37			42	85	−1.8	30
2878	B8	5.43	10.7	64	257	− 4	0.1	270		138	0	360						
2867	B3	5.09	10.7	63	257	− 4	1.4	291		138	+ 21	30			50	60	−1.4	53
3151	B3	4.98	12.0	63	265	− 1	0.6	288		128	+ 36	68						
3802	B2	5.42	14.8	62	285	− 3	1.1	207		112	− 13	41			31	72	−2.2	22
3668	B3	4.10	14.2	46	286	+13	1.2	228		104	+ 5	24			32	45	−3.4	27
4405	B1	3.51	17.3	56	302	−12	1.3	189		100	− 2	17			34	32	−3.9	26
4180	B5	4.71	16.3	47	303	+ 1	1.0	217	+ 7.0*	94	+ 15	73	+ 1.4		26	1.27	−3.2	21
4496	B5	5.90	17.7	54	305	−14	1.3	347		96	(+160)	39		Excl.				
4288	B	5.34	16.8	42	311	+ 1	1.2	194	−18.0*	86	− 3	32	−21.0		31	56	−2.2	24
4274	Oe	5.37	16.8	41	311	+ 1	1.0	101		86	(− 96)	35		Excl.				
4642	B5	5.42	18.3	44	317	−15	1.6	142		86	− 38	24			34	0.55	−1.9	29
4334	B1p	4.87	17.0	34	319	+ 4	0.7	82	+ 8.6	78	−113	36	+ 8.1					
4369	B3	5.50	17.2	33	321	+ 2	0.9	63		76	−132	44						
4697	B3	5.38	18.4	33	329	−12	0.9	153		74	− 25	37						

LIST 3—*Continued*

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal. Long	Gal. Lat.	100 μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0″.0001	$\frac{r_\pi}{\pi}$	M	π_2 Unit 0″.0001
4560	Oe5	5.86	18h0	24°	334°	− 2°	0.6	180°		66°	− 4°	43°						
4549	B	5.73	17.9	23	335	− 1	1.2	215		65	+ 30	23			28	0.50	−2.0	26
4604	B8pc	4.01	18.1	21	338	− 3	0.6	141	− 7.0†	64	− 42	18	− 3.3					
4612	Ba	5.42	18.2	21	338	− 3	0.5	169	Bin.	64	− 14	36						
4613	B1	6.02	18.2	20	339	− 2	1.0	180		62	− 3	23			30	41	−1.6	21
4668	B8	6.03	18.4	18	341	− 4	0.7	207		60	+ 28	60						
4793	B8	5.89	18.8	23	340	−12	1.6	202		65	+ 28	17			40	37	−1.1	32
4865	B3	5.41	19.0	19	345	−13	0.1	90	Bin.	62	− 80	344						
4934	B8p	4.58	19.3	16	350	−15	0.5	217	−10.0‡	60	+ 51	39	− 5.2					
5240	B8	5.20	20.4	18	353	−30	1.6	142	−18.3	67	− 13	8	−15.5		45	20	−1.5	41
4548	B5c	3.92	17.9	3§	358	+12	1.4	172	− 4.0	39	− 15	10	+ 5.9		57	0.27	−2.3	48
4527	B8	5.73	17.9	1§	355	+12	0.6	45	−17.2‡	42	−142	50	− 7.9					

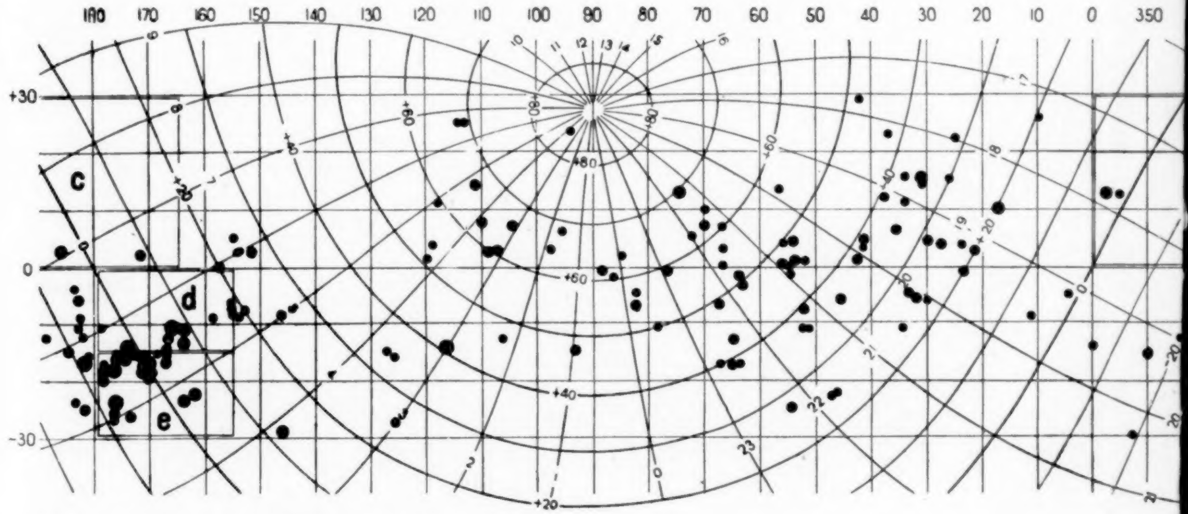
* 1 observation.
† Velocity center of mass from orbit.
‡ Spect. Binary, estimated velocity of center of mass.
§ Declination north.

GRONINGEN
January 1914



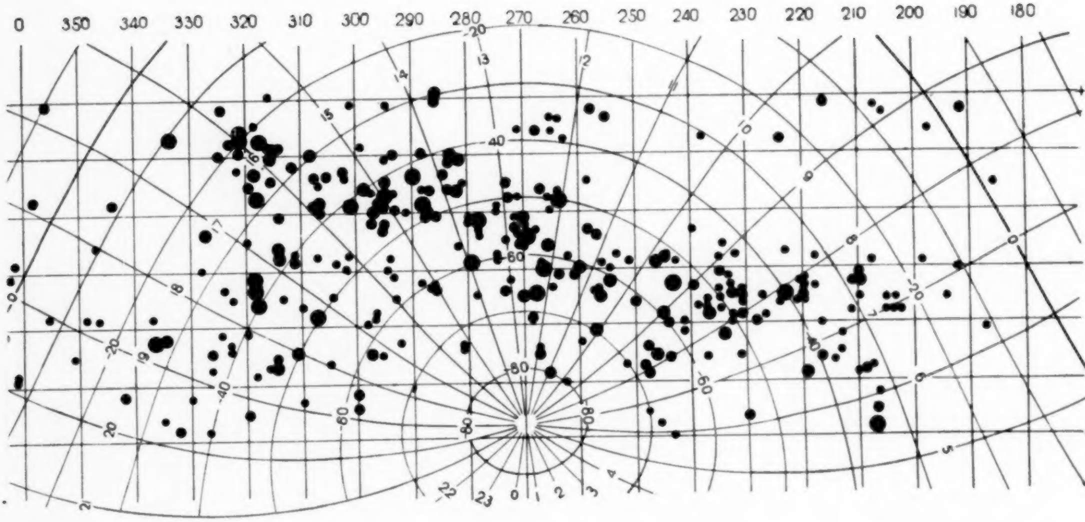
MAP 1.—HELIUM STARS HAVING TOTAL P.M.≧0.″017

● BRI



MAP 2.—HELIUM STARS HAVING TOTAL P.M.≦0.″016

ATE II



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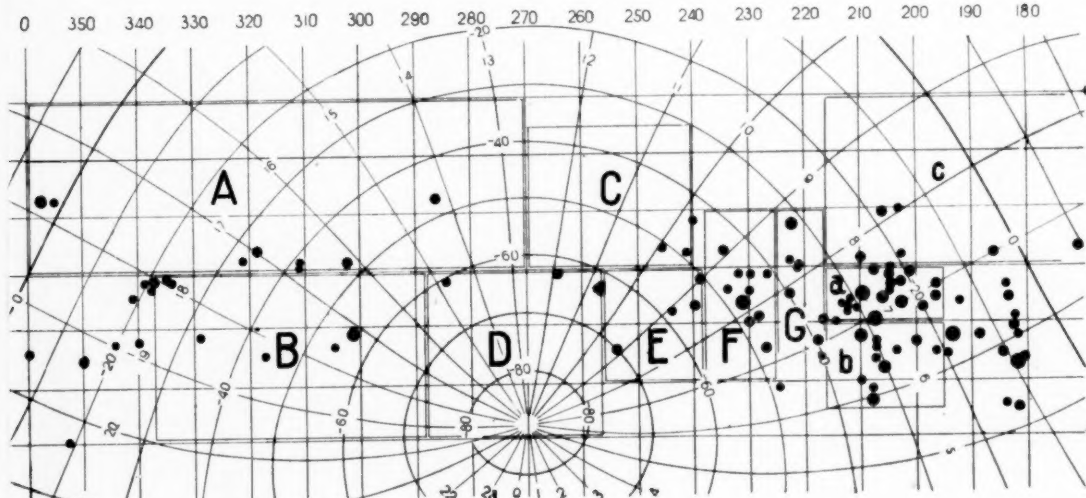

IMPROVEMENTS IN THE OPTICAL SYSTEM OF THE STELLAR SPECTROGRAPH

By J. S. PLASKETT

It has long been my opinion that the dense silicate flint, the O 102 glass of the Jena Glass Works, which has been almost exclusively chosen as the prism material for the stellar spectrograph, is too highly colored and hence too absorbing in the violet for the best results. This opinion was confirmed by my experiments with the grating spectrograph[1] and also by some work with objective prisms of light flint glass. The O 102 is probably the most suitable among the dense flints and was apparently chosen because the high dispersion and resolving power, together with the great compactness and symmetry of form, demanded in the modern three-prism stellar spectrograph, seemed most easily satisfied by the use of dense flint glass.

It was not, however, at first recognized that the conditions required in one-prism instruments were not the same. Their principal usefulness has proved to consist in obtaining the spectra of early-type stars, where the lines are few in number, are each due in general to one element only, and are often broad and diffuse. Under such circumstances high dispersion and resolving power are not necessary and indeed are, when the lines are diffuse, a disadvantage. Furthermore, to increase the measureable material in spectra with few lines, it is desirable that as long a range of wave-length as possible be photographed at one exposure. Under these conditions O 102 glass is not suitable as it is not only highly dispersive but it is highly absorbing in the violet and ultra-violet, thus considerably limiting the range of wave-lengths available.

The only investigation bearing on the absorption of glasses suitable for prism material seemed to be the work undertaken at Potsdam by Vogel, Müller, and Wilsing[2] in their investigation of

[1] *Astrophysical Journal*, **37**, 373, 1913.

[2] *Berichte der Berliner Akademie*, November 1896.

the materials proposed for the 80-cm objective and the spectrographs to be used with it. Their values in the photographic region for three glasses, O 203 Ordinary Silicate Crown, O 340 Ordinary Light Flint, and O 102 Heavy Silicate Flint, are given in Table I. An approximate idea of the absorptions of other glasses can be obtained by comparing their relative dispersions in different parts of the spectrum. Consequently in the same table are given the dispersion constants of four other glasses selected from the Jena list as being the most promising.

TABLE I

CONSTANTS OF REPRESENTATIVE GLASSES

Kind of Glass	Trade No.	n	Δ	Ratios of Dispersion			Transmissions through 10 cm for Wave-Length				
				α	β	γ	4341	4000	3950	3900	3750
Ordinary Silicate Crown........	O 203	1.5175	0.00877	0.642	0.702	0.568	0.667	0.695	.....	0.583	0.583
Baryta Light Flint.........	O 722	1.5797	.01087	.632	.707	.577	.....	.....	.....	.....	.....
Baryta Light Flint.........	O 1266	1.6042	.01381	.616	.711	.594	.....	.....	.....	.....	.....
Ordinary Light Flint.........	O 340	1.5774	.01396	.614	.713	.600	.569	.614	.....	.456	.388
Baryta Flint.....	O 748	1.6235	.01599	.605	.713	.604	.....	.....	.....	.....	.....
Ordinary Silicate Flint.........	O 93	1.6245	.01743	.604	.715	.609	.....	.....	.....	.....	.....
Heavy Silicate Flint.........	O 102	1.6489	0.01919	0.600	0.714	0.615	0.502	0.463	0.167	0.025	.....

In the table n is the index of refraction for D, Δ is the dispersion from C to F, and α, β, γ the ratios of the dispersions between A′ and D, between D and F, and between F and G′ to the dispersion Δ. The last five columns give the experimentally determined values of the transmission for a thickness of 10 cm of the glass. The transmission of the other glasses is presumably intermediate between the given values, varying probably with the run of the relative dispersions, with α and γ.

The transmission through 10 cm of O 102 glass shows how unsuitable it is for prism material, not only on account of the general absorption all along the photographic spectrum but especially on account of the strong absorption around K. The O 722 glass was

selected as the most promising, as it gives considerably greater dispersion than the O 203 with probably very little more absorption.

Consequently a prism of fine annealed glass of Baryta Light Flint O 722 was ordered from the J. A. Brashear Co. In order that it could be placed in the one-prism spectrograph without alterations of the frame, the deviation of the ray at minimum must be 60°. As it was proposed to carry the measurements farther to the violet, the central wave-length was chosen as λ 4200, midway linearly between λ 4550 and λ 3934 or between λ 4862 and λ 3750. As the constants of the material of which the prism was to be made were $n=1.5782$, $\Delta=0.01078$, $\alpha=0.637$, $\beta=0.708$, $\gamma=0.576$, even more favorable for transparency than the tabular values, the angle required was 68° 57′, the length of the sides for a 51 mm (2 inches) aperture was 118 mm (4.64 inches) and of the base 133 mm (5.25 inches). The prism was made 57 mm (2.25 inches) high and is consequently a large block of glass. It is beautifully colorless and transparent, and, notwithstanding its large size, shows no trace of imperfect annealing. Careful tests by diaphragming different sections have shown that every part defines equally well and as a dispersing piece it is practically perfect.

In order to obtain the full advantage of the transparency of this glass, the isokumatic collimator objective, whose central component is decidedly yellowish, was replaced by a Brashear triplet and a triplet camera objective of slightly longer focal length than the "Single Material" was also obtained. In order to reduce the loss by reflection in the internal surfaces, which, as will be seen later, is a serious matter, amounting to over 30 per cent for the two objectives, it was desirable to cement them. Watch oil and glycerine were first tried but, though they answer admirably at ordinary temperatures, when the spectrograph gets much below freezing, they become crystallized or mottled in appearance and cannot be used. Some special balsam prepared for cementing together three-color transparencies which remains softer than the ordinary balsam was successfully used and up to the present no ill effects on the definition have appeared.

The spectrograph box was dismounted and the new prism and objectives installed and carefully tested with an artificial source

before being used on the stars. The definition given was excellent and some preliminary comparative tests of the two prisms showed striking advantages in efficiency, especially in the violet, of the light flint. A curious change in the character of the field given by the triplet camera was noticed, for, while with the dense flint prism the field was concave to the lens, with the light flint it was convex and of smaller curvature. With a prism of intermediate dispersion the field would probably be flat. The curvature of course is small, the difference between the center λ 4200 and the ends λ 4550 and K being about 0.1 mm. By accommodating, the focus of no part of the spectrum need be more than 0.05 mm from the plate. The dispersion given by the combination is 54.5 A per millimeter at H_γ, 48.3 A at λ 4200, and 37.5 A at K. This is almost exactly three-fifths of the linear dispersion given with the O 102 prism and can be made equal by increasing the focal length of the camera, although this is not readily accomplished with the present arrangement.

Before giving the results of the comparative tests of exposures, it will be of interest to obtain the intensities of the emergent pencils by computation. The intensities after reflection at the various surfaces are obtained from the well known formulae used for this purpose, while the intensities after absorption have been computed for the prisms, from measured and interpolated values of O 102 glass, and from interpolated values of O 722 glass. For the isokumatic collimator, its absorption was estimated by comparing its color with that of the O 102 prism. It was found that between 50 and 60 mm thickness of the latter gave about the same depth of color as the isokumatic, and its absorption was hence assumed as equal to that of the prism. The absorption of the material of the other objectives was taken as the same as that of O 722 glass. The mean thickness of the O 102 prism is 57 mm, of the O 722, 66 mm, of the triplet lenses about 20 mm, and of the single material 10 mm. The intensities, after losses by reflection and by absorption, and the final emergent intensities are given in the tables below.

The total emergent intensities, for the various wave-lengths in Table III, show that the new optical system transmits a considerably larger percentage of the incident light than the old, giving

a pencil 50 per cent stronger in the blue, 75 per cent stronger at H_δ, and four times as strong at K. The comparatively small

TABLE II

INTENSITIES AFTER REFLECTIONS

System	Optical Part	First Reflection Air to Glass	Reflection at Each Inner Surface	Method	Final Intensity
Old	Isokumat Collimator	0.9535	0.99	$0.9535^2 \times 0.99^4$	0.8735
	O 102 Prism	.874	....	....	.779
	Single Material Camera	.9535	.9535	$.9535^4$	.8247
	Total	....	....	....	.5607
New	Triplet Collimator	.9535	.99	$.9535^2 \times 0.99^4$	.8735
	O 722 Prism	.868	....	....	.768
	Tiiplet Camera	0.9535	0.99	$0.9535^2 \times 0.99^4$	.8735
	Total	....	....	....	0.5861

TABLE III

INTENSITIES AFTER ABSORPTIONS

System	Optical Part	H_β	4600	H_γ	4200	H_δ	4000	*K*	H_ζ
Old	Isokumat Collimator	0.842	0.761	0.696	0.658	0.658	0.639	0.400	0.108
	O 102 Prism	.842	.761	.696	.658	.658	.639	.400	.108
	Single Material Camera	.983	.974	.960	.945	.945	.945	.925	.900
	Total intensity after absorption	.697	.564	.464	.408	.408	.385	.148	.011
	Total emergent intensity	.391	.316	.260	.229	.229	.216	.083	.006
New	Triplet Collimator	.973	.962	.938	.925	.925	.925	.903	.884
	O 722 Prism	.916	.881	.809	.775	.775	.783	.714	.666
	Triplet camera	.973	.962	.938	.925	.925	.925	.903	.884
	Total intensity after absorption	.858	.814	.712	.664	.664	.671	.582	.520
	Total emergent intensity	0.503	0.477	0.417	0.389	0.389	0.393	0.341	0.305

values show also how large a percentage of the light is lost in the optical system of even the most efficient form of spectrograph, and

how important it is to look after apparently minor details. For example, cementing the triplet camera makes the difference between transmission of 0.8735 and 0.7515, a gain of 16 per cent, while cementing both gives increased transmission of 35 per cent. Similarly the loss by absorption in the isokumatic collimator is some 30 per cent greater than in the triplet, and this computed loss is fully borne out by the experimental results.

Experimental tests of the relative efficiencies of the new and old optical systems have been carried out in two ways: first, by comparing a number of plates of the same stars by the two systems, making allowance for the differences in seeing; second, by making direct comparative tests following one another on the sun and stars in exactly the same way as with the grating spectrograph.[1] The latter method gives results probably more reliable and certainly more directly comparable than the former. By the first method three spectra of one star with the new system, with exposures in the ratio of 1, 2, 3, were made side by side on one plate and this was repeated for a number of stars. Each of these plates was then compared with about ten plates of the same stars with the old spectrograph, and it was comparatively easy to get reliable estimates of the relative exposures. Similarly, by the second method, seven or eight exposures, with times increasing about 50 per cent on each, were made of the same stars by each form of the instrument, each set of exposures being on one plate and all the plates developed together. It is evident that numerous accurate comparisons of the relative intensities at any wave-length can easily be made from such a set of exposures.

Table IV contains a summary of the mean values obtained by both methods. In (6) of this table we get the relative exposures experimentally determined for the new and old optical systems, and it will be seen how marked a saving in exposure time is effected. The new system requires less than two-fifths the exposure of the old at H_γ, one-quarter at H_δ and only one-ninth at K. It must not be forgotten, however, that the dispersion of the new system is only three-fifths that of the old and to make them directly comparable the figures in (6) should be multiplied by five-thirds.

[1] *Astrophysical Journal*, **37**, 373, 1913.

This has been done in Table V, giving the relative experimentally determined efficiencies. If we divide the total emergent intensities of Table III we get the relative computed efficiencies.

TABLE IV

MEAN VALUES OF RELATIVE EXPONENTS AT EIGHT WAVE-LENGTHS

Method	No.	Optical Systems Compared, Last Unity	H_β	4600	H_γ	4200	H_δ	4000	K	H_ζ
First	1	O 722 Triplet:O 102 Isokumat.	0.381	0.402	0.363	0.285	0.225	0.156	0.108	0.059
Second . .	2	O 722 Triplet:O 102 Triplet.	.570	.554	.471	.395	.313	.263	.177	.123
	3	O 102 Triplet:O 102 Isokumat.	.785	.754	.726	.712	.712	.687	.638	.586
	4	O 722 Triplet:O 102 Isokumat.	.508	.501	.406	.351	.270	.180	.115	.080
	5	O 722 Triplet:O 102 Isokumat (2)×(3)	.447	.418	.342	.281	.223	.181	.113	.072
	6	Mean of (1), (4), (5)	0.445	0.440	0.370	0.306	0.239	0.172	0.112	0.070

TABLE V

RELATIVE EXPERIMENTAL AND COMPUTED EFFICIENCIES

	H_β	4600	H_γ	4200	H_δ	4000	K	H_ζ
Experimental Old : New	0.742	0.733	0.617	0.510	0.398	0.287	0.187	0.117
Computed Old : New	0.770	0.666	0.624	0.589	0.589	0.550	0.245	0.019

There is good agreement in general in these figures and the deviation between λ 4200 and λ 4000 is probably due chiefly to insufficient data as to the absorptions of the glasses in this region. It is my opinion that the run of the absorptions in the O 102 glass must be much more gradual than given in the tables and furthermore that the absorption of the isokumatic collimator may be such as to account for part of this difference. A more gradual run of the absorptions would make excellent agreement between observed and computed values. Another factor will influence the magnitude of the experimental values for certain wave-lengths, the fact that all wave-lengths of the light forming the star image are not in sharp focus on the slit. Owing to the color-curve of objective and correcting lens there is a difference of over 3 millimeters in the position

of the star focus for H_γ light and for light at H_β and K. The slit was placed so that wave-lengths about λ 4000 and λ 4650 were in focus on it and the higher experimental value at λ 4600 may be due to this cause.

It may be pointed out that (3) in Table IV gives relative exposures when triplet and isokumat collimator are interchanged and that they agree well with the estimated values of the absorption of the isokumat in Table III until we get near K. Here evidently the absorption of the borosilicate flint of the isokumat differs from that of the O 102, producing a more gradual change. Both sets of figures form striking evidence of the unsuitability of this objective for stellar spectrographs.

In addition to the gain in efficiency, another great advantage of the new optical parts is the uniformity in the intensity of early-type spectra. With the O 102 prism, if the exposure was such as to make the K line measureable, the region around H_γ was so much overexposed as to block up the fainter metallic lines. Furthermore, spectra from the light flint prism contain four or five more measureable hydrogen lines than those made with the dense flint, thus considerably increasing the material available for measurement in stars with few lines.

The results of this investigation may be summarized as follows:

1. The dense silicate flint (O 102) glass, almost universally used as prism material in stellar spectrographs, has been shown to be too highly absorbing all along the photographic spectrum and especially toward and in the ultra-violet for the best results in radial velocity work. This is especially the case in single-prism spectrographs employed on early-type stars.

2. The substitution of a baryta light flint prism, O 722, for the O 102 has diminished the exposure times (when both are reduced to the same linear dispersion) by 22 per cent at H_γ, 48 per cent at H_δ, and 70 per cent at K, besides giving considerably more measureable material in the ultra-violet without overexposure around H_γ.

3. The substitution of a Brashear Triplet for the isokumat collimator objective has, owing to the strong absorption of the latter, effected a further saving of about 30 per cent.

4. The cementing of the contact curves of the triplet collimator and camera objectives diminishes the loss by reflection over what would occur with uncemented lenses by 35 per cent.

5. The ratios of the intensities of the emergent pencils from the old and new optical parts are 0.62 at H_γ, 0.40 at H_δ, and 0.19 at K. The actual ratios of the exposures required owing to the three-fifths dispersion of the new prism are 0.37 at H_γ, 0.24 at H_δ, and 0.11 at K. Although the smaller dispersion will probably entail proportionally larger probable errors, the fact that nearly all the stars within range of the present equipment have been observed, and that the new optical parts enable stars at least a magnitude fainter to be reached, form partial compensation for the diminished accuracy.

6. Finally, the investigation has shown the importance in stellar spectroscopy, where the light is always meager in quantity, of so selecting the materials and designing the optical parts of a stellar spectrograph that all the losses by reflection and absorption may be minimized; and the results indicate what a great saving in exposure time and consequent increase in output and range may be effected.

It gives me pleasure to acknowledge here the readiness of the director, Dr. W. F. King, to supply the apparatus needed in this investigation, as well as the interest he has taken in the work.

Since the above was written, measures of the transparency to violet and ultra-violet light of five optical glasses have been kindly sent me by the Jena Glass Works. These measures with the corresponding refraction-constants are given in the following tables.

The measures show that only a very general guide to the relative absorptions can be obtained from the ratios of the dispersions and this only when glasses of similar composition are considered. The ordinary silicate flints are evidently more suitable than the baryta flints for prism material, as O 118, which is recommended by the Jena people, has apparently less absorption than O 722 and at the same time 50 per cent greater dispersion. The choice seems to lie between this material and the ultra-violet flint, depending upon how far into the ultra-violet one wishes to go. It is a simple matter from the accompanying data to compute the dimensions of the prisms and the losses by reflection and absorption for any given dispersion and spectral region and hence to determine the most suitable material.

TRANSMISSION OF GLASSES

Wave-Length	U.V. 3248 Ultra-Violet Flint		O 722 Baryta Light Flint		O 748 Baryta Flint		O 340 Ordinary Light Flint		O 118 Ordinary Flint		O 919 Ordinary Flint
	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.
4250..	......	......	......	......	......	......	......	......	......	......	0.96
4050..	......	......	0.99	0.86	0.98	0.84	0.98	0.83	0.99	0.88	......
3970..	0.98	0.96	......	......	......	......	......	......	......	......	.94
3840..	.98	.87	......	......	......	......	......	......	......	......	0.86
3650..	......	......	.87	.25	.81	.11	.86	.22	.90	.33	......
3610..	.98	.79	......	......	......	......	......	......	......	......	......
3460..	.92	.45	......	......	......	......	......	......	......	......	......
3340..	......	......	0.39	0.00	0.22	0.00	0.35	0.00	0.43	0.00	......
3250..	0.78	0.08	......	......	......	......	......	......	......	......	......

REFRACTION CONSTANTS

Kind of Glass	n	Δ	ν	Ratios of Dispersion		
				α	β	γ
U.V. 3248 ultra-violet flint....	1.5332	0.00964	55.4	0.634	0.705	0.573
O 722 baryta light flint......	1.5797	.01078	53.8	.632	.707	.577
O 748 baryta flint...........	1.6235	.01599	39.1	.605	.713	.604
O 340 ordinary light flint.....	1.5774	.01396	41.4	.614	.713	.600
O 118 ordinary flint.........	1.6129	.01660	36.9	.606	.713	.607
O 919 ordinary flint.........	1.6315	0.01770	35.7	0.600	0.715	0.613

Dominion Observatory
Ottawa
February 1914

ON THE PRESSURE-SHIFT OF THE LINES OF THE ZINC SPECTRUM AT LOW PRESSURES

By V. F. SWAIM

INTRODUCTION

Previous studies of pressure-effect have, for the most part, been at comparatively high pressures. A part of the earlier work of Humphreys and Mohler[1] and of Humphreys[2] dealt with pressures as low as 4 atmospheres, but the more recent work of Humphreys[3] has been at much higher pressures. Similarly, the work of Duffield[4] upon the arc spectra of iron, gold, and silver contains some results obtained at pressures as low as 5 atmospheres, but the majority of his values are also for high pressures. The work of Rossi[5] upon the arc spectra of titanium and vanadium is based principally upon pressures between 25 and 100 atmospheres.

All the work referred to above showed that the lines of any spectrum were shifted toward the red as the pressure about the source was increased; also, that the amount of this shift for a given increase of pressure was greater for the long wave-lengths than for the short ones.

Later work by Gale and Adams[6] showed that many lines of the spectrum of iron could be classified into groups, according to their behavior under pressure; also, that the displacement for the lines of each group varied as the third power of the wave-length.

St. John and Ware[7] found that a certain group of iron lines was shifted toward the violet under pressure. This result was later confirmed by Gale and Adams.[8] Goos[9] has shown that the value of the wave-length of many of the iron lines depends on the part

[1] *Astrophysical Journal*, **3**, 114, 1895.

[2] *Ibid.*, **4**, 249, 1896; **6**, 169, 1897.

[3] *Ibid.*, **22**, 217, 1905; **26**, 18, 1907.

[4] *Ibid.*, **26**, 375, 1907; *Phil. Trans.*, A **208**, 111, 1908.

[5] *Proc. Roy. Soc.*, A **83**, 414, 1910.

[6] *Astrophysical Journal*, **35**, 10, 1912.

[7] *Ibid.*, **36**, 14, 1912.

[8] *Ibid.*, **37**, 391, 1913.

[9] *Ibid.*, **38**, 2, 1913.

of the arc used and on the strength of the current. Many of the iron lines show large displacements, when exposures near the negative pole are compared with exposures at the middle of the arc.

The interpretation of the pressure-shifts of spectrum lines, the relation of pressure-shift to wave-length and to other properties of the elements, is not at all evident. Any theory which is to explain the facts as they are known today must permit both a positive and a negative shift. As will be shown in this investigation, it must permit either an increasing or a decreasing displacement as the wave-length decreases. The wave-length of many lines is strikingly affected by current and by the portion of the arc used. This may mean that wave-length is a function of temperature. As a first step toward the formulation of a satisfactory theory of pressure-shifts we must know more about the behavior of lines of different substances, especially series lines.

In view of the work of Gale and Adams, together with the fact that previous work upon elements which have series lines has been done at high pressures, and not with a constant current, it seemed desirable to examine the series lines of some element, at low pressures, paying special attention to the points brought out by Goos and St. John. Hence the work described in this article has a twofold purpose: (1) to see whether a classification of the lines into groups according to their behavior under pressure coincides with their series classification; (2) to determine the relation between the wave-length and the displacement per atmosphere for the series lines of some element. Zinc was chosen as the element to be studied in this investigation.

APPARATUS AND ADJUSTMENTS

The grating used was a 6-inch Rowland concave of 21.5 feet radius, ruled with 15,000 lines to the inch. It was mounted in the usual way, as fully described by Ames.[1]

The pressure box used by Gale and Adams[2] at the Pasadena Laboratory of the Mount Wilson Observatory was used in this experiment. A quartz window was substituted for the glass window in the front of the brass hood, through which the light from

[1] *Phil. Mag.* (5), **27**, 369, 1889. [2] *Astrophysical Journal*, **35**, 10, 1912.

the horizontal arc passes. The light passing through this window was focused upon the vertical slit by means of a quartz lens placed so as to give a threefold magnified image. This image was kept the same width across the slit throughout the experiment. Special care was taken to see that the cone of light covered the grating with a good margin. To provide for accurate guiding during an exposure, when the arc shifts from one point to another upon the electrodes, the simple expedient of placing the lens upon a slide rack was adopted. However, the horizontal motion of the arc, due to the burning-away of the poles, was cared for by moving the pressure box horizontally. Hence the lens was never moved, except vertically.

The arc was produced by a 110-volt direct circuit, between horizontal brass poles, 7 mm in diameter. Several exposures were made in which the current and pressure were varied in order to determine the following points: (1) the greatest distance which the poles may be separated, and yet give a steady arc; (2) the largest values of the pressure and current which will give the lines of the series, narrow and distinct.

It was decided to use a 2.5-mm arc at 3.5 amperes throughout this entire investigation. Great care was taken to see that all exposures were made exactly upon the center of the arc. For the lines of the second subordinate series the maximum range of pressure was approximately from 5 cm to 2.2 atmospheres. For the first subordinate series the maximum range of pressure was from 5 cm to 5 atmospheres. However, many plates were taken at pressures considerably below the maximum. All these plates showed the displacement of a particular line to be directly proportional to the pressure within this range.

The camera took 2×19-inch plates and was provided with a shutter of the form usually used with the concave grating.

METHOD OF PHOTOGRAPHING

For the purpose of accurate comparison it was necessary to obtain side by side photographs of the spectrum of the substance in question as given by the arc under the two pressures used. It was also necessary to guard as far as possible against any accidental

movement of any part of the apparatus, and to be able to detect with certainty the exact amount of such a displacement, should it occur.

The first of these requirements was met by means of the shutter, mentioned above. In nearly every case the middle strip was exposed to the arc under the low pressure, after which air was pumped into the pressure box, the shutter rotated, and strips of the plate above and below the first exposure were exposed to the arc under the higher pressure.

The second requirement was satisfied by photographing, on some part of the plate, a few lines at atmospheric pressure, before and after both vacuum and pressure exposures. By this order of exposures, one is able to detect an accidental displacement, should it occur either during an exposure of the zinc lines or between the exposures of the zinc lines. In order to make these exposures separate from the lines for which the shift was being investigated, black paper curtains were hung in front of the camera, shielding the parts of the plate on which the respective lines appeared. Hence, either set of lines could be photographed by raising the corresponding curtain. There were generally lines appearing upon the plate other than those whose displacement was being determined, in which case they were used to determine the accidental displacement. However, if no such lines appeared, iron poles were substituted for the brass poles and the iron lines used to determine the accidental displacement.

The length of exposure for both the comparison and the pressure spectrum on Seed 27 and Cramer Crown plates was as follows: group IIN3, 20 seconds; group IN4, 2 minutes; groups IIN4, IIN5, and IN5, 20 minutes; groups IN6 and IIN6, 1.5 hours; non-series lines, from 10 to 15 minutes.

METHODS OF MEASURING

The plates were carefully measured with a small comparator constructed by William Gaertner & Co. The instrument reads directly to thousandths of a millimeter, and may be estimated to ten-thousandths.

The majority of the plates were taken in the second order, where the dispersion was a little more than 1 mm per angstrom. In

all about two hundred photographs were taken and the displacements determined from those lines whose positions were well defined by reason either of their sharpness or of their reversals. Each plate was measured twice, once in each direction. In each measurement, five settings were made upon the vacuum line and ten settings upon the pressure line.

EXPERIMENTAL RESULTS

The extraordinary variety in behavior of different spectrum lines under pressure has of course been noted by all observers.

The first photographs showed that all the lines of the second subordinate series of the zinc spectrum broadened very unsymmetrically toward the red, under high pressure, thus making the determination of their displacement a matter of personal judgment. This broadening was much greater for the violet lines than for the blue ones. For instance, the lines λ 2712.60 and λ 2684.29 could hardly be said to be more than blurs under a pressure of 5 atmospheres, while the lines λ 4810.71, λ 4722.66, and λ 4680.38 were fairly good for measurement. Hence, the lines of the second subordinate series of the zinc spectrum will be placed in class five of the Gale and Adams[1] classification of spectrum lines, with the added statement that the violet lines explode completely under high pressure. However, it was further found that by using pressures below 2.5 atmospheres, the position of the real line appeared, and that at a pressure of 1.5 atmospheres the time of exposure could be so regulated that the real line appeared with but very little shading on its side.

The lines of the first subordinate series of the zinc spectrum showed a tendency toward unsymmetrical broadening at the above pressures, as only one reversal appeared until a pressure of approximately 5 atmospheres was reached. The one reversed line was λ 2802.11 which appears in group IN5. It reverses nicely at a pressure of 1.25 atmospheres, while the other members of that group showed no tendency toward reversing, even at 5 atmospheres. The lines λ 3345.13 and λ 3302.67 reversed very nicely at a pressure of approximately 5 atmospheres, but the other members of

[1] *Astrophysical Journal*, **35**, 10, 1912.

group IN4 only became very broad and slightly unsymmetrical. The close triple and the close double lines in group IN5 could not be obtained as single lines (except λ 2802.11) at pressures above 1.2 atmospheres and then the two lines λ 2800.90 and λ 2771.05 were very diffuse. However, the other two lines were fairly good and afforded very good measurements upon a plate from which some of the most accurate measurements of group IIN4 were taken. In general, the lines of the first subordinate series broaden immensely under pressure, showing no tendency toward reversal at moderate pressures, thus making the measurement of their displacement very difficult. Most of the lines of this series belong to class five of the Gale and Adams classification.

We may make the general statement that the violet lines are broader and much more diffuse than the blue lines in both the first and second subordinate series of zinc.

A number of special photographs, some of which were exposed for 1.5 hours, showed that it was impossible to photograph many of the lines of the zinc spectrum for the purpose of this investigation.

In Table I are given the results of the measurements of the displacements of the zinc lines in the spectrum of the arc between brass poles, taken within the range of pressures mentioned above, and reduced to one atmosphere. The wave-length of the line according to Kayser and Runge is given in the first column; in the second column is given the classification of the zinc lines under a pressure of 5 atmospheres, according to the Gale and Adams classification of spectrum lines; in the third column, the series group as given by Kayser and Runge; the mean displacement per atmosphere, in the fourth column; the number of plates measured, in the fifth column; and the displacements as found by Humphreys with a difference of pressure of 7 atmospheres (except λ 3075.99, and it was taken at a difference of 6 atmospheres) is given in the sixth column.

The groups IIN3 and IN4 were measured first, and found to agree pretty closely with the values obtained by Humphreys. However, it was a great surprise to find that the displacements of group IIN4 were larger than those for group IIN3. This was wholly unexpected, and much doubt was felt as to its validity.

Several times each and every adjustment of the apparatus was gone over, and new photographs taken, but each time with the same displacements. After taking about fifty photographs, all

TABLE I

Wave-Length	Class	Group	Mean Δ per Atm.	No. of Plates	Humphreys 7 Atm. Difference
Second Subordinate Series					
4810.71	5	IIN3	0.0099	4	0.056
4722.26	5	IIN3	0.0098	4	0.051
4680.38	5	IIN3	0.0099	4	0.061
3072.19	5	IIN4	0.0163	15	0.049
3035.93	5	IIN4	0.0165	15	0.044
3018.50	5	IIN4	0.0160	9	0.046
2712.60	5	IIN5	0.0204	4	....
2684.29	5	IIN5	0.0199	4	....
2670.67	5	IIN5	0.0176	3	....
2567.99	5	IIN6	0.0224	3	....
First Subordinate Series					
3346.04	5	IN4	0.0046	3	0.030
3345.62	5	IN4	0.0045	3	0.025
3345.13	2	IN4	0.0047	3	0.026
3303.03	5	IN4	0.0044	3	0.022
3302.67	2	IN4	0.0044	3	0.030
3282.42	5	IN4	0.0045	3	0.027
2802.11	2	IN5	0.0024	3	....
2800.90	5	IN5	0.0082	3	....
2800.17	5	IN5	0.0088	3	....
2771.05	5	IN5	0.0085	3	....
2770.94	5	IN5	0.0077	3	....
2756.53	5	IN5	0.0092	3	....
2608.65	5	IN6	0.0112	3	....
2582.57	5	IN6	0.0091	3	....
2570.00	5	IN6	0.0085	3	....
Non-Series Lines					
6362.58	4	....	0.0300	3	....
4630.06	3	....	0.0120	3	....
4058.02	2	....	0.0057	3	....
3740.12	3	....	0.0084	3	....
3572.90	3	....	0.0085	3	....
3075.99	2	....	0.0021	3	0.0120*

* Difference of 6 atmospheres.

of which appeared to have about the same displacement when reduced to one atmosphere, fifteen of the best plates were measured, and the mean adopted as correct. Since the groups IIN4, IIN5,

and IN_5 all appeared upon the same plate and were subjected to the same instrumental corrections, it seems impossible that there shou.d be any mistake concerning the relation of the displacements for these groups to one another. Also, it is wholly impossible that such a mistake should lie in the judgment of the maximum of the lines, since group IIN_4 appeared very sharp and narrow up to approximately 1.25 atmospheres. The individual settings on this group could be made within one division on the micrometer head. However, the groups IIN_5 and IN_5 were not quite so good.

Table I shows a very good agreement between the values of the displacements obtained in this experiment and those obtained by Humphreys at higher pressures, with the exception of the group IIN_4. Of course it is possible that there should be no difference in the values of the displacements obtained under the different experimental conditions, but it seems more probable that the value of the current and the part of the arc used for the exposures should be the source of difference in the two experiments.

It is to be noted that the displacements for all the lines of a particular group of a series is nearly a constant for a given pressure (except λ 2802.11). If the mean wave-length for each group of the second subordinate series be plotted against the reciprocal of the mean displacement for that group, one finds that the points fall almost exactly upon a straight line as shown in Fig. 1. The corresponding numerical values are given in Table II.

TABLE II

Mean Wave-Length	Group	$1/\Delta$
4738	IIN_3	101
3042	IIN_4	61
2689	IIN_5	52
2568	IIN_6	45

This graph (Fig. 1) shows that the displacement of the lines of the second subordinate series vary inversely with the wave-length, i.e., the displacement increases as the wave-length decreases. Very little weight should be given to the line λ 2567.99, since it was very diffuse and could hardly be measured at all.

If the mean wave-length for each group of the first subordinate series be plotted against the reciprocal of the cube root of the mean displacement for that group, it is found that the points fall upon a straight line, as shown in Fig. 2. The corresponding numerical values are given in Table III. This graph (Fig. 2) shows that the displacement of the lines of the first subordinate series varies

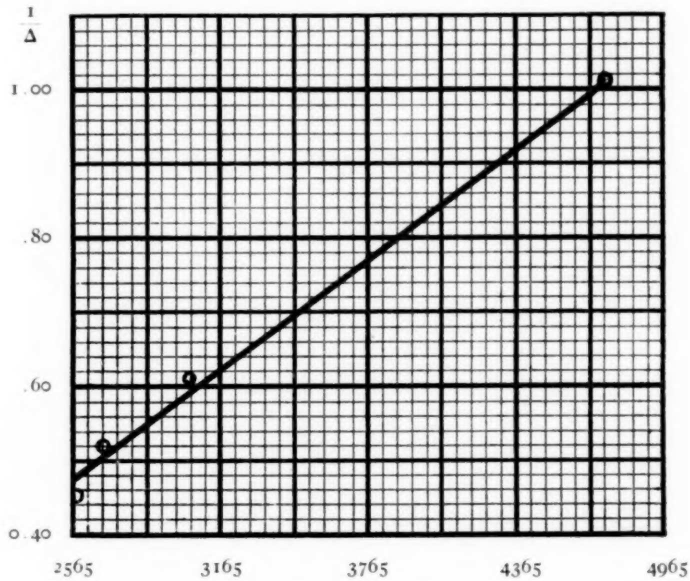

FIG. 1.—Second subordinate series

inversely with the cube of the wave-length. In making this graph, the line λ 2802.11 was not included. This line has such a different value for its displacement, and reverses so much more easily than the other lines of group IN5, that it seems probable that it should not be considered as a member of this series. Hence, it will be classed as a non-series line in this article.

TABLE III

Mean Wave-Length	Group	$1/\sqrt[3]{\Delta}$
3321	IN4	61
2780	IN5	49
2587	IN6	46

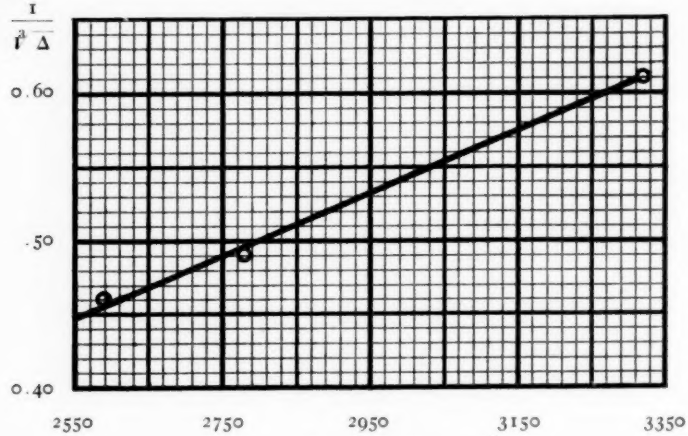

Fig. 2.—First subordinate series

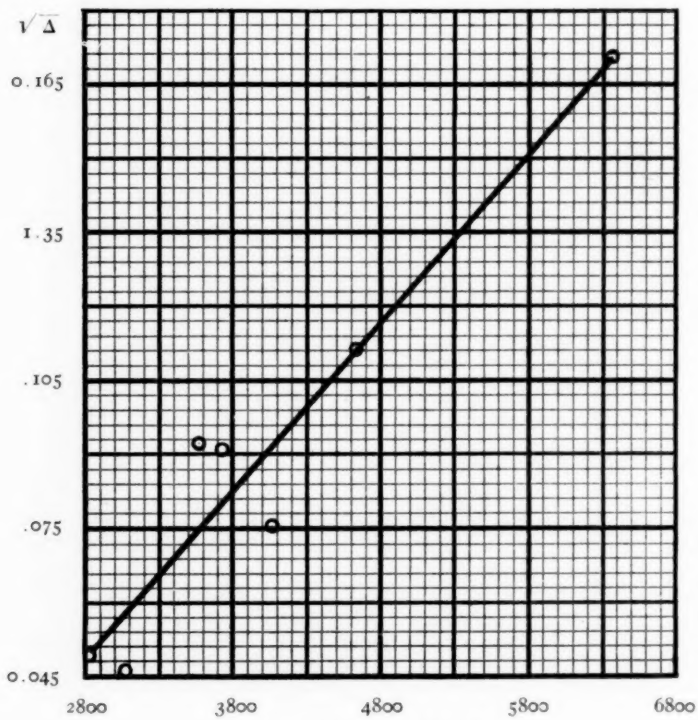

Fig. 3.—Non-series lines

All the foregoing results seem to indicate a connection between the diffuseness of the line and its susceptibility to pressure.

If we plot the wave-lengths of the non-series lines against the square root of their respective displacements, we obtain the graph as shown in Fig. 3. The corresponding values are given in Table IV. Fig. 3 shows that these displacements are well represented by an expression involving the square of the wave-length.

TABLE IV

Wave-Length	Group	$\sqrt{\Delta}$
6363	Non-series	.17
4630	"	.11
4058	"	.075
3740	"	.091
3573	"	.092
3076	"	.046
2802	"	.049

It is very striking that the displacements of the lines of the different series should vary inversely according to different powers of the wave-length, while the displacements of the non-series lines vary directly according to a still different power of the wave-length.

SUMMARY OF RESULTS

1. The classification of the lines according to their action under pressure agrees with their series classification (except λ 2802.11).
2. The displacement of the lines of the second subordinate series varies inversely with the wave-length, at a particular pressure.
3. The displacement of the lines of the first subordinate series varies inversely with the cube of the wave-length, at a particular pressure.
4. The displacement of the non-series lines varies directly with the square of the wave-length, at a particular pressure.
5. The line λ 2802.11 is better classified as a non-series line.

In conclusion, the author desires to express his appreciation to the staff of the Physics Department for their kindly interest in this work and especially to Professor Gale, at whose suggestion and under whose direction the investigation was carried out.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
April 4, 1914

THE EFFECT OF SELF-INDUCTION ON THE NITROGEN BANDS

By E. P. LEWIS

In a previous article[1] some effects produced by self-induction on the band spectrum of nitrogen were described. It was found that it enhanced the negative bands and caused them to appear in all parts of the tube. Hemsalech[2] had previously found that at atmospheric pressure self-induction caused these bands to appear prominently in the spectrum of the spark discharge between electrodes of certain metals. Recently, in the course of a study of the effects of changed conditions on the vacuum tube spectrum of nitrogen, other effects were noted which had escaped observation in 1903 on account of the small dispersion of the system used and the absorption of the ultra-violet by the glass prism. The quartz spectrograph previously described[3] was used. Evident changes were produced in the first group of positive bands by self-induction, but on account of the small dispersion in this region attention was concentrated on the second positive group. As before noted, self-induction caused the appearance of the negative bands everywhere, but more striking was the marked change produced in the structure of the positive bands. The general character of these changes is shown in Fig. 1, Plate III, and is most clearly seen in the group of bands with the first head at λ 3371. Comparing the spectrum of the simple discharge (*a*) with that when self-induction and capacity are used (*b*), it is seen that these effects are: (1) relative enhancement of the more refrangible lines belonging to the principal head, so that they may be easily followed to the head of the next group at λ 3159; (2) almost complete suppression of the subheads of the group and the lines belonging to them; (3) increased sharpness of the lines. Observations were made with the light from the capillary of a quartz tube, with end-on glass tubes with quartz windows ranging in diameter from 1 to 20 mm; and with a spark gap 1 cm

[1] *Astrophysical Journal*, **17**, 258, 1903.

[2] *Comptes rendus*, **132**, 1040, 1901.

[3] *Astrophysical Journal*, **23**, 390, 1906.

long in a tube of 2 cm diameter. At high pressures it was noted that the effects produced by self-induction were somewhat increased by increase in self-induction; but at low pressures substantially as great an effect was produced by 50 turns as by 500 turns in the self-induction coil. At high pressures, from about 30 cm up, the effects were almost as marked with the simple discharge as with self-induction. At pressures above 35 cm (spark gap) or 10 cm (capillary) capacity alone produced a line spectrum; at lower pressures, bands, which showed in lesser degree the same characteristics as those due to self-induction. The spectrum of the discharge from a small Tesla coil at different pressures was similar to that of the simple discharge, the lines rapidly dying out in going from the head, and each subhead of a group was prominent. When the quartz tube was used, the capillary was heated to bright red, and it was found that this produced slight effects similar to those of self-induction.

No record of effects of such magnitude as those described here has been found by the writer, although effects of a similar nature have been described. Hagenbach and Konen[1] and Stark[2] found that the bands of nitrogen became more extended toward the violet as the pressure or temperature increased. Berndt[3] found, conversely, that at low pressures the bands became more concentrated near the heads, while at atmospheric pressure they were extended toward the violet by capacity and self-induction, with improved resolution.

Inasmuch as the current density, potential gradient, and temperature must have varied widely under the different conditions employed, it seems unlikely that any one of these factors alone can determine the effects most prominently shown when self-induction is employed. It would seem more probable that the cause must be looked for in some characteristic peculiar to self-induction, perhaps in connection with the oscillation period. Ladenburg[4] shows that not only is the logarithmic decrement of the oscillations decreased by self-induction, but that the current density is diminished. He explains the narrowing of lines by self-induction as the result of

[1] *Phys. Zeit.*, **4**, 227, 1903.

[2] *Ibid.*, **7**, 357, 1906.

[3] *Ann. der Physik*, **7**, 946, 1900.

[4] *Ibid.*, **38**, 248, 1912.

a reduced number of emission centers due to this cause. In the present case, however, observations taken with very small simple currents and the still smaller currents due to the Tesla discharge show quite the opposite effect from that due to self-induction—a narrowing of the bands, poorer resolution, and relative increase in the intensity of the subheads.

Fig. 1, Plate III, is the reproduction of a photograph (enlarged about three times) taken at a pressure of about 6 mm. The lines of the band beginning at λ 3371 can be distinctly followed to the head of the next group at λ 3159. The subheads of the first group are very faint and the lines belonging to them make scarcely any impression on the film. The heads of the negative bands at λ 3298 and λ 3296 show as a slight shading. On the less refrangible side of λ 3371 some lines can be seen which belong to a band which appears only when hydrogen or ammonia vapor is present, and which is further described on p. 154. Some of the films taken showed no trace of any of these bands, owing to slight hydrogen impurity, but this film was chosen as most suitable for reproduction. Near the head the lines of the band are not resolved.

Inasmuch as this band may be followed much farther toward the violet than in any case previously described, it seemed of interest to find whether Deslandres' formula applies. The wave-lengths of the lines were determined by Hartmann's interpolation formula, the iron spark being used for comparison. The wave-lengths of the comparison lines are those of the International System as given by Kayser in Vol. 5 of the *Handbuch der Spektroscopie*. In this region the values are about 0.14 of an angstrom less than those of the Rowland system. The wave-numbers were computed from the formula $n=n_0+Am^2$, where $n_0=296{,}617$ and $A=1.817$. Hermesdorf[1] has measured many lines of this band. For comparison, the third decimal in his results was dropped and each wave-length reduced by 0.14. Those corrected values which approximate most closely to the calculated values are given in Table I. There are at least twice as many more lines in his table which do not at all fit the computed values; so that it seems probable that there are several series in this band, all but the principal one being suppressed by self-induction.

[1] *Op. cit.*, 11, 161, 1903.

TABLE I

m	λ	$n=\frac{1}{\lambda}\times 10^9$ Obs.	Calc.	Difference	Hermesdorf (Corrected) n	λ
0	3371.35	(Head)	296,617		296,617	3371.35
1			619			
2			624			
3			633		296,640	71.08
4			646			
5			662		677	70.69
6			683			
7			706		728	70.08
8			733			
9			764	−15	779	69.50
10			799	2	797	69.31
11			837	− 1	838	68.84
12			879	3	876	68.40
13			924	−21	945	67.62
14			973	− 3	976	67.27
15			297,026	− 5	297,032	66.63
16			082	7	075	66.15
17			143	9	132	65.50
18			206	− 2	208	64.64
19			273	− 8	281	63.82
20			344	−12	356	62.97
21	3362.40	297,407	418	11, 21	397	62.50
22	61.50	487	496	9, 20	476	61.61
23	60.51	574	578	4, 12	566	60.69
24	59.50	664	664	0, 22	642	59.73
25	58.50	752	753	1, 22	731	58.73
26	57.50	840	846	6, 26	820	57.69
27	56.46	933	942	9, 21	921	56.59
28	55.29	298,036	298,042	6, −12	298,054	55.00
29	3354.10	298,142	298,146	4, − 8	298,154	3353.96
30	52.87	243	252	9, 22	230	53.11
31	51.70	356	363	7, − 7	370	51.54
32	50.49	463	477	14, − 7	484	50.26
33	49.16	582	595	13, − 6	601	48.95
34	47.79	704	716	12, − 8	724	47.59
35	46.40	828	842	14, − 2	846	46.21
36	45.00	954	971	17, − 2	973	44.78
37	43.44	299,093	299,104	11, 0	299,104	43.31
38	41.80	240	240	0, − 2	242	41.77
39	40.30	374	380	6, 1	379	40.24
40	38.70	515	524	9		
41	37.05	661	671	10		
42	35.41	814	822	8		
43	33.66	970	976	6		
44	32.00	300,120	300,135	15		
45	30.20	282	295	13		
46	28.30	452	464	12		
47	26.45	621	632	11		
48	24.50	797	805	8		
49	22.50	978	982	− 4		
50	20.49	301,160	301,159	− 1		
51	18.50	341	345	4		

TABLE I—*Continued*

m	λ	$n=\frac{1}{\lambda}\times 10^9$ Obs.	$n=\frac{1}{\lambda}\times 10^9$ Calc.	DIFFERENCE	HERMESDORF (CORRECTED) n	HERMESDORF (CORRECTED) λ
52	16.40	532	532	0		
53	14.34	719	717	− 2		
54	12.26	908	915	7		
55	10.10	302,105	302,114	9		
56	7.85	311	319	8		
57	5.53	524	524	0		
58	3303.26	302,731	302,732	1		
59	3200.85	952	942	−10		
60	98.54	303,164	303,158	− 6		
61	96.22	378	378	0		
62	93.71	602	601	− 1		
63	91.20	841	829	−12		
64	88.70	304,072	304,060	−12		
65	86.20	303	294	− 9		
66	83.60	544	532	−12		
67	81.00	783	774	− 9		
68	78.32	305,034	305,019	−15		
69	75.66	282	274	− 8		
70	72.96	538	520	−13		
71	70.15	796	790	− 6		
72	67.49	306,045	306,036	− 9		
73	64.60	316	300	−16		
74	61.82	377	567	−10		
75	58.90	852	837	−15		
76	55.98	307,127	307,124	− 3		
77	53.05	404	395	− 9		
78	50.10	683	673	−10		
79	47.14	963	957	− 6		
80	44.05	308,257	308,246	−11		
81	40.90	556	539	−17		
82	37.83	850	834	−16		
83	34.70	309,148	309,133	−15		
84	32.55	450	438	−12		
85	28.38	753	745	− 8		
86	25.18	310,060	310,055	− 5		
87	3221.93	310,373	310,370	− 3		
88	18.62	692	688	− 4		
89	15.28	311,015	311,009	− 6		
90	11.92	340	335	− 5		
91	08.61	660	661	1		
92	05.20	993	996	3		
93	01.70	312,334	312,332	− 2		
94	3198.12	684	673	−11		
95	94.66	313,022	313,015	− 7		
96	91.07	365	363	− 3		
97	87.57	719	715	− 4		
98	83.98	314,072	314,067	− 5		
99	80.40	427	426	− 1		
100	76.75	787	787	0		
101	73.06	315,154	315,152	− 2		
102	69.38	519	521	2		
103	65.64	892	894	2		
104	62.00	316,257	316,270	13		

PLATE III

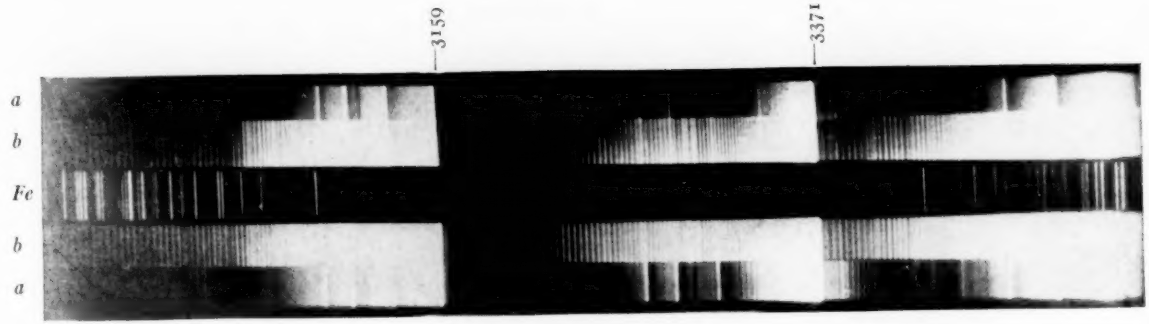

Fig. 1

Fig. 2

The differences between the observed and the calculated values of the wave-numbers are, except in a few instances, well within the limits of errors of measurement; but it may be significant that the corrections are almost uniformly positive at the two ends of the series and negative in the middle. This may indicate a systematic error in the determination of wave-lengths, possibly due to a warping of the film in drying; or it may indicate that the Deslandres formula does not apply with exactness.

Thanks are due to Mr. S. L. Quimby for assistance in measuring the film and calculating wave-lengths.

University of California
March 30, 1914

THE ULTRA-VIOLET BAND OF AMMONIA

By E. P. LEWIS

Eder[1] describes the spectrum of the flame of ammonia burning in oxygen. A number of bands in the ultra-violet attributed by him to ammonia have since been found to be the well-known third group of "nitrogen" bands of Deslandres, and are probably due to an oxide of nitrogen. A band between the green and red observed by Eder has likewise been described by other investigators. Another band in the ultra-violet, extending from λ 3295 to λ 3432, was held by Eder to be the principal band of ammonia, but apparently it has not been described by others. Kayser[2] says: "Whether this band belongs to our third group [of nitrogen] is unknown." Recently, while studying the spectra of mixtures of nitrogen and hydrogen in vacuum tubes, I found this band at all pressures up to about 70 cm, and it becomes specially prominent when self-induction is used. It is found with all proportions of the two gases, from the time that the red line of hydrogen first appears until the nitrogen bands disappear. With nitrogen free from hydrogen or hydrogen free from nitrogen it is not found. Small traces of oxygen have no effect upon it, while considerable quantities destroy it. It seems to require the presence of these two gases alone, and might reasonably be attributed to ammonia.

There is, however, such a marked difference between the conduct of this band and of that between the green and red that further examination seems desirable. When ammonia gas is introduced into the vacuum tube the visible band is at first very prominent, but it rapidly dies out, whatever may be the nature or the intensity of the discharge. It can be maintained only by allowing a continuous stream of gas to pass through the tube. It would appear from this that the ammonia is rapidly dissociated by the current. The ultra-violet band is likewise found in the spectrum of the ammonia gas, but it is most persistent, and still appears after the

[1] *Denkschriften Wiener Akad.*, **60**, 1, 1893.

[2] *Handbuch der Spektroscopie*, **5**, 833.

discharge has been passed for hours. This strongly suggests that different emission centers are responsible for the two bands; either a different and more stable compound of hydrogen and nitrogen, or possibly a combination of these two gases with some impurity present in the tube, such as traces of oxygen or hydrocarbons.

A photograph of this band is reproduced in *d*, Fig. 2, Plate III; *c* is the comparison spectrum of nitrogen. The nitrogen contained about 10 per cent of hydrogen, the pressure was 2 mm, and the band was made more prominent by the use of self-induction. Its appearance is complicated by superposition over the nitrogen band at λ 3371. The resolution is better than that obtained by Eder, but not sufficient to make it worth while to attempt to separate the lines of this band from those of nitrogen. An early attempt will be made to secure a photograph of this spectrum with larger dispersion, as this band is so peculiar in structure that it seems desirable to make a more detailed study of it.

UNIVERSITY OF CALIFORNIA
March 30, 1914

Minor Contributions and Notes

NOTE ON RADIAL MOVEMENT IN SUN-SPOTS

In his very valuable contribution to the study of radial movement in sun-spots, published in the *Astrophysical Journal* for June 1913, Mr. St. John states that the usual course of the displaced lines over spots in a solar image 170 mm in diameter "shows no sharp break and the displacement does not suddenly cease at the periphery of the penumbra, but the line gradually returns to its normal course."

This differs from the statement I have published, viz., that there is "an appreciable break or jolt in the lines at the points where they pass from the penumbrae on to the surrounding photosphere."[1]

The deduction from my plates would appear to be that the motion outward ceases abruptly just at the point where the maximum velocity is obtained, and Mr. St. John's observation "seems to remove one great difficulty in explaining the displacement as due to motion."

The question is of course to some extent one of degree, and it is of importance to determine how far outside the penumbral limits the motion can be traced. It appears to me that the photographic evidence is by no means conclusive, because of the inevitable movements of the solar image on the spectrograph slit while the plate is being exposed; and I am inclined to believe from my own results that the limiting distance within which the velocity apparently changes from its maximum value to zero may be very much smaller than is implied in the sentence I have quoted from Mr. St. John.

I have found that the clearness with which the jolt in the lines is brought out in the photograph depends mainly on the exposure time, but also on the state of the "seeing" during the exposure and on the accuracy of guiding. With very short exposure times the guiding factor does not come in appreciably, and if the seeing is

[1] *Monthly Notices of the Royal Astronomical Society*, January 1910, p. 219.

reasonably good the limits of umbra and penumbra in the photographed spectrum will be very clearly defined. In such plates the displacements are greater and end more abruptly than in plates which have had a long exposure, especially if guiding has been necessary and the seeing has been poor. The reason for this is sufficiently obvious, for with long exposures the unsteadiness of the image on the slit tends to spread the displacements at each point over an appreciable length of the lines, and this has the effect both of reducing the amount of the maximum displacement and of spreading the displacement beyond the point of maximum outside the penumbra.

As I am at the present time on a visit to North India I am unable to refer to my records, but I believe that in my best plates the exposure times did not exceed about 30 seconds; and in those plates especially in which the displacements were first detected the definition is unusually fine, the limits of umbra and penumbra being very clearly marked. In these the displacements certainly seem to end exactly at the dividing line between penumbra and photosphere, and I should say from memory that the movement could not be traced so far outside the penumbra as one-fiftieth of the diameter of the spot.

It is not perhaps fair to judge of the definition of Mr. St. John's plates from the reproductions given in his paper, but it is almost inevitable from the form of spectrograph used that the exposures must have exceeded mine many times over, and Mr. St. John himself states that "with the utmost possible care in guiding the slit cannot be . . . . rigorously held upon the same point with respect to the edge of the penumbra." The Mount Wilson plates have an advantage over mine in the somewhat greater linear dispersion, and the much larger scale of the spot image, but I think this may be more than offset by the longer exposure times. Evidence of the longer exposures seems to be discoverable in the generally smaller values of the maximum radial motion which Mr. St. John's results show when compared with my own.

In the Kodaikanal spectrograph high dispersion is obtained by inclining the camera to a high angle with the collimator, actually 60°, and not by the use of a very high-focus lens. This method

magnifies the spectrum in one dimension only, viz., dispersion, instead of in two dimensions as with increased focal length. The intensity of the spectrum, therefore, is considerably greater than in an autocollimating spectrograph giving the same linear dispersion.

It is of great interest to settle the question of the limits of the radial motion. If it is eventually found that the displacements really end abruptly at the edge of the penumbra, the difficulty of the sudden stoppage of the motion might be explained by supposing the photosphere surrounding the spot to be heaped up, so that the reversing layer is at a higher level outside the spot. The moving gases would then be hidden at the point where they would penetrate the raised photosphere.

J. EVERSHED

KASHMIR
August 15, 1913

As Mr. Evershed says, the question of how suddenly the motion outward from spots and tangential to the solar surface passes from its maximum value to zero is of much importance. In the case of the high-level calcium vapor our observations agree in attributing a smooth curve to the displacement of the K_3 line,[1] there being no "jolt or break" at the outer boundary of the penumbra; and in respect to the lines of the lower reversing layer, the apparent difference between our observations is to some extent one of degree. I remark that "in the type of vortex in which the data of the present paper seemed to find their interpretation, there is an enormous flux of energy from the vortex into and below the reversing layer. The mechanical energy of motion is rapidly transformed into other forms, as the velocity decreases rapidly with distance. The question of how rapidly will be taken up in a later investigation";[2] while the deduction from Mr. Evershed's plates, especially those taken under the most favorable observing conditions, is that the motion outward ceases abruptly just at the point where the maximum velocity is attained.

[1] *Monthly Notices*, **70**, 220.

[2] *Mount Wilson Contr.*, No. 69, p. 29; *Astrophysical Journal*, **37**, 350, 1913.

There is force in the point to which Mr. Evershed calls attention, that of necessity the exposures were longer with the instrument employed at Mount Wilson, in fact, from two to six times as long as he used, and hence any unsteadiness of the solar image and any irregularity in guiding would tend to smooth out an abrupt break in the course of the line. The spectrum was often observed visually with the slit in position for maximum effect and even then no abrupt change in the course of the line was detected.

In the program arranged for the approaching spot maximum, it has been planned to make a special examination of this question and particularly of the variation of velocity with distance from the center of the spot and to determine how far the line displacement can be traced beyond the boundary of the penumbra. In doing this, a simple and apparently very efficient guiding device suggested by Mr. Evershed will be used, and it is hoped that with the large image given by the 150-foot telescope and by taking advantage of the best seeing, a more definite determination can be made of the limits to which the velocity can be traced and of the rate of change of this velocity, if it is found that the displacements do not end abruptly at the outer boundary of the penumbra. If, as Mr. Evershed suggests, a stoppage of the motion is due to the heaping-up of the photosphere around the spot, it might well be that the suddenness of the stoppage would vary with different spots, the piling-up of the photosphere depending upon the solar activity. It is to be noted that Mr. Evershed made his observations at a time of great solar activity and was able to observe more spots than I did, and that those I did observe appeared at a time when the solar activity was nearing its minimum.

Charles E. St. John

Mount Wilson Solar Observatory
November 13, 1913

Reviews

Annuaire astronomique et météorologique pour 1914. Par Camille Flammarion. Paris: Ernest Flammarion. Pp. 427. Figs. 125. Fr. 1.50.

It is no small distinction to have edited an astronomical annual for half a century, and it is a pleasure to offer our congratulations to the ever-active editor on the appearance of this handy little volume for the fiftieth year. It contains all the astronomical data for the year in a popular presentation, with small maps of movements of the planets, and numerous pictures of these bodies, charts of the constellations for each month, special notices on recent discoveries, a review of the meteorology of the year preceding, and a chapter on wireless telegraphy. May M. Flammarion long continue this excellent means of popularizing science!

F.

Milton's Astronomy. The Astronomy of Paradise Lost. By Thomas N. Orchard. London: Longmans, Green & Co., 1913. Pp. 288, with six plates and seven figures. $2.50 net.

This interesting book will serve a useful purpose in two directions: in giving to the "general reader" a brief history of astronomy and in introducing him to the beauty of Milton's poetry through numerous quotations. It will be useful also as a reference book for classes in astronomy and in literature for its exposition of the status of astronomical knowledge in Milton's time and of his cosmology as found in *Paradise Lost.*

A word must be added as to the illustrations. The subjects chosen are admirable, but it is greatly to be regretted that better half-tones were not secured to do them justice and to conform to the otherwise high quality of the volume.

F. B. L.